



DETAILED SOLUTIONS AND CONCEPTS - LOGARITHMIC FUNCTIONS
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Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

Definition of Logarithmic Function

The logarithmic function is the inverse of the exponential function $y = b^x$, that is $x = b^y$.

Solving the inverse of the exponential function for y , the logarithmic function is written as

$$y = \log_b x \quad (\text{pronounced } \mathbf{\log \textit{base } b \textit{ of } x}), \text{ where } \mathbf{b} \text{ is a positive number other than } \mathbf{1}$$

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

Characteristics of Graphs of Logarithmic Functions

Most logarithmic functions $f(x) = \log_b x$ and their transformations are best graphed with a graphing utility because the y-values get extremely large/small very quickly and are difficult to show in a hand-drawn *Cartesian Coordinate System*.

The graph of $f(x) = \log_b x$ has the following shapes:



- The graph consists of a SMOOTH curve with a rounded turn.
- Logarithmic functions and their transformations have *vertical asymptotes*.
- The equation of the *vertical asymptote* of the graph of $f(x) = \log_b x$ is $x = 0$, which is the x-axis.

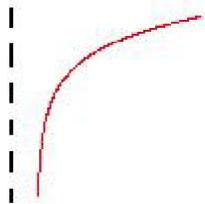
- ONLY horizontal shifts of the graph of $f(x) = \log_b x$ change the equation of the *vertical asymptote*.
- The graph is never parallel to the x-axis, but moves away from it at a steady pace.
- The graph is never parallel to the *vertical asymptote*, but moves toward it at a steady pace.
- The graph has a distinct concavity, which, depending on a transformation, can be concave up or down.
- There is always an x-intercept.
- There is at most one y-intercept. This means that some graphs may have no y-intercept, while others may have one.

Problem 1:

Find the following for $h(x) = \log x$.

- Domain
- Coordinates of the x-intercept
- Coordinates of the y-intercept
- Equation of the vertical asymptote

The graph has the following shape:



- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$$x > 0$$

Therefore, in *Interval Notation*, the domain is $(0, \infty)$.

- Coordinates of the x-intercept:

$$0 = \log x$$

and changing to exponential form, we find

$$x = 10^0$$

$$x = 1$$

The coordinates are $(1, 0)$.

- Coordinates of the y-intercept:

But any logarithm of 0 is undefined. Therefore, we can conclude that this function has NO y-intercepts.

- Equation of the Vertical Asymptote:

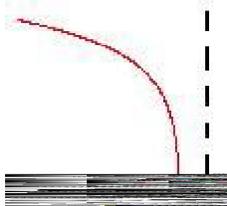
Since our equation is of the form $y = \log_b x$ with $b = 10$, the y-axis is the vertical asymptote whose equation is $x = 0$.

Problem 2:

Find the following for .

- Domain
- Coordinates of the x-intercept
- Coordinates of the y-intercept
- Equation of the vertical asymptote

The graph has the following shape:



- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$$-x > 0$$

Multiplying both sides by -1, we find

$$x < 0$$

Therefore, in Interval Notation, the domain is $(-\infty, 0)$.

- Coordinates of the x-intercept:

The coordinates are $(-1, 0)$.

- Coordinates of the y-intercept:

But any logarithm of 0 is undefined. Therefore, we can conclude that this function has NO y-intercepts.

- Equation of the Vertical Asymptote:

This is a reflection of $\log(x)$ about the y-axis. Horizontal shifts of $\log(x)$ affect the location of the vertical asymptote. Reflections about the y-axis and horizontal stretches/shrinks affect the location of the vertical asymptote indirectly because they affect the horizontal shift.

Since we have no horizontal shift, the reflection about the y-axis does not affect the equation of the vertical asymptote, which is still $x = 0$.

Problem 3:

Find the following for $f(x) = \log(-x)$.

- Domain
- Coordinates of the x-intercept
- Coordinates of the y-intercept
- Equation of the vertical asymptote

The graph has the following shape:

- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$$x + 2 > 0$$

$$x > -2$$

Therefore, in Interval Notation, the domain is $(-2, \infty)$.

- Coordinates of the x-intercept:

The coordinates are $(8, 0)$.

- Coordinates of the y-intercept rounded to 2 decimal places:

The coordinates are $(0, -0.70)$.

- Equation of the Vertical Asymptote:

This is a horizontal shift of 2 units to the left and a vertical shift of 1 unit down. Since horizontal shifts affect the location of the vertical asymptote, and we do have a horizontal shift 2 units to the left, the equation of the vertical asymptote becomes $x = -2$. Vertical shifts DO NOT affect it.

Problem 4:

Find the following for $f(x) = \frac{1}{x+2} - 1$.

- Domain
- Coordinates of the x-intercept. Round to 2 decimal places.
- Coordinates of the y-intercept
- Equation of the vertical asymptote

The graph has the following shape:

- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$$x > 0$$

Therefore, in *Interval Notation*, the domain is $(0, \infty)$.

- Coordinates of the x-intercept rounded to 2 decimal places:

The coordinates are $(0.01, 0)$.

- Coordinates of the y-intercept:

But any logarithm of 0 is undefined. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

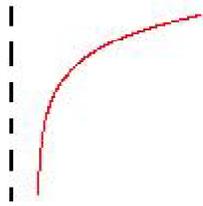
This is a vertical shift of 2 units upward. Vertical shifts **DO NOT** affect the location of the vertical asymptote. Therefore, the equation of the vertical asymptote is still $x = 0$.

Problem 5:

Find the following for $f(x) = \ln(x + 2)$. Note we are discussing the natural logarithm "el en of x"!

- Domain
- Coordinates of the x-intercept
- Coordinates of the y-intercept
- Equation of the vertical asymptote

The graph has the following shape:



- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$$x > 0$$

Therefore, in *Interval Notation*, the domain is $(0, \infty)$.

- Coordinates of the x-intercept:

$$0 = \ln x$$

$$e^0 = x$$

$$x = 1$$

The coordinates are $(1, 0)$.

- Coordinates of the y-intercept:

$$g(0) = \ln 0$$

But any logarithm of 0 is undefined. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

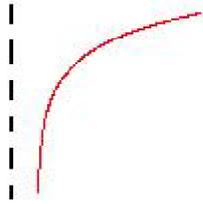
Since our equation is of the form $y = \log_b x$ with $b = e$, the y-axis is the vertical asymptote whose equation is $x = 0$.

Problem 6:

Find the following for $p(x) = \log_2(x - 1) + 3$.

- Domain
- Coordinates of the x-intercept
- Coordinates of the y-intercept
- Equation of the vertical asymptote

The graph has the following shape:



- Domain:

The domain of the logarithmic function consists of all numbers that DO NOT make the y-value imaginary (log of a negative number) or undefined (log of zero).

We do this by finding the numbers that make the argument greater than 0. That is,

$$x - 1 > 0$$

$$x > 1$$

Therefore, in *Interval Notation*, the domain is $(1, \infty)$.

- Coordinates of the x-intercept:

$$0 = \log_2(x - 1) + 3$$

$$\log_2(x - 1) = -3$$

$$x - 1 = 2^{-3}$$

$$x = \frac{1}{8} + 1 = 1\frac{1}{8} = \frac{9}{8}$$

The coordinates are $(\frac{9}{8}, 0)$.

- Coordinates of the y-intercept:

$$p(0) = \log_2(0 - 1) + 3 = \log_2(-1) + 3$$

But any logarithm of a negative number is imaginary. Therefore, we can conclude that this function has **NO** y-intercepts.

- Equation of the Vertical Asymptote:

This is a horizontal shift of $y = \log_2 x$ by **1** unit to the right and a vertical shift of **3** units up. Since horizontal shifts affect the location of the vertical asymptote, and we do have a horizontal shift **1** unit to the right, the equation of the *vertical asymptote* becomes $x = 1$. Vertical shifts **DO NOT** affect the location of the vertical asymptote.