DETAILED SOLUTIONS AND CONCEPTS - INVERSE TRIGONOMETRIC FUNCTIONS

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PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

You might recall from algebra that a function can only have an inverse if it is *one-to-one*. According to the *Horizontal Line Test*, in order for a function to be *one-to-one* no horizontal line drawn though the graph of the function can intersect with the graph more than once. Obviously, knowing the graphs of trigonometric functions, they do not pass the *Horizontal Line Test*. However, we can "restrict" the domain of any function so that it passes this test.

Definition of Inverse Trigonometric Functions

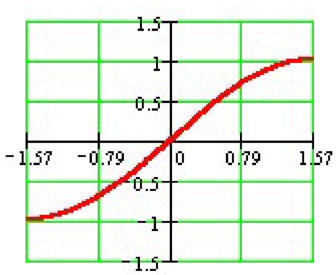
Be sure to memorize the domains and ranges of each inverse trigonometric function!
Remember that in the case of inverse functions, the domain of one is the range of the other!

Restricted Sine Function

$y = \sin x$

Domain: $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Range: $-1 \le y \le 1$



Inverse Sine Function

If we exchange x and y in the restricted sine function, we get $x = \sin y$, which is rewritten as

$$y = \arcsin x \text{ or } y = \sin^{-1} x$$

Both are pronounced either ***arcsine of x*** or ***sine inverse of x***!

Domain: $-1 \le x \le 1$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Restricted Cosine Function

$y = \cos x$

Domain: $0 \le x \le \pi$

Range: $-1 \le y \le 1$

Inverse Cosine Function

If we exchange x and y in the restricted cosine function, we get $x = \cos y$, which is rewritten as

$$y = \arccos x \text{ or } y = \cos^{-1} x$$

Both are pronounced either ***arccosine of x*** or ***cosine inverse of x***!

Domain: $-1 \le x \le 1$

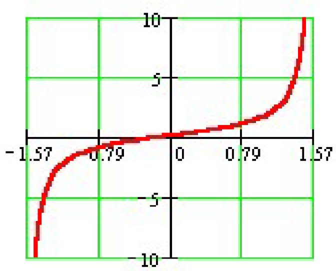
Range: $0 \le y \le \pi$

Restricted Tangent Function

$y = \tan x$

Domain: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Range: ***All Real Numbers***



Inverse Tangent Function

If we exchange x and y in the restricted tangent function, we get $x = \tan y$, which is rewritten as

$$y = \arctan x \text{ or } y = \tan^{-1} x$$

Both are pronounced either ***arctangent of x*** or ***tangent inverse of x***!

Domain: $-\infty < x < \infty$

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Cotangent, Secant, and Cosecant Functions

For completeness, the definitions of the inverse cotangent, secant, and cosecant functions are included. **Please note that the ranges of the inverse secant and cosecant function are not universally agreed upon!**

$y = \text{arccot}\, x$ or $y = \cot^{-1} x$

Domain: $-\infty < x < \infty$ Range: $0 < y < \pi$

$y = \textbf{arc sec } x$ or $y = \sec^{-1} x$

Domain: $|x| \geq 1$ Range: $0 < y < \pi$, $y \neq \frac{\pi}{2}$

$y = \textbf{arc csc } x$ or $y = \csc^{-1} x$

Domain: $|x| \geq 1$ Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $y \neq 0$

Problem 1:

Using a calculator, find ***sin*** $^{-1}$ ***(0.86)*** in degrees and radians rounded to two decimal places.

You access ***sin*** $^{-1}$ by first pressing the "2nd" or "Shift" key on your calculator and then pressing the "sin" key.

If you want your angles to be in radians, make sure your calculator is in "Radian Mode". If you want your angles to be in degrees, your calculator must be in "Degree Mode."

This calculation will result in approximately 59.32^o or ***1.04*** radians.

Problem 2:

Using a calculator, find ***cos*** $^{-1}$ ***(-0.87)*** in degrees and radians rounded to two decimal places.

You access ***cos*** $^{-1}$ by first pressing the "2nd" or "Shift" key on your calculator and then pressing the "cos" key.

This calculation results in approximately 150.46^o or ***2.63*** radians.

Problem 3:

Using a calculator, find ***tan*** $^{-1}$ ***(0.59)*** in degrees and radians rounded to two decimal places.

You access ***tan*** $^{-1}$ by first pressing the "2nd" or "Shift" key on your calculator and then pressing the "tan" key.

This calculation results in approximately 30.54^o or ***0.53*** radians.

Problem 4:

Using a calculator, find ***cot** $^{-1}$ **(-0.3541)*** in degrees and radians rounded to two decimal places.

Since there is no calculator key to evaluate the inverse cotangent, we must use the inverse tangent as follows.

Find ***tan** $^{-1}$ **(1/-0.3541)*** which equals approximately ***-70.50°*** or ***-1.23*** radians.

However, since the range of the inverse cotangent function is restricted to $0 < y < \pi$, we need a second quadrant angle which is

$\theta \approx$ ***180° - 70.50° = 109.50°*** or $\theta \approx \pi$ ***- 1.23 = 1.91*** radians

Therefore, ***cot** $^{-1}$ **(-0.3541)*** is approximately ***109.50°*** or ***1.91*** radians.

Problem 5:

Using a calculator, find ***sec** $^{-1}$ **(-1.43)*** in degrees and radians rounded to two decimal places.

Since there is no calculator key to evaluate the inverse secant, we must use the inverse cosine as follows.

Find ***cos** $^{-1}$ **(1/-1.43)*** which equals approximately ***134.37°*** or ***2.35*** radians.

This is in the range $0 < y < \pi$ of the inverse secant function!

Therefore, ***sec** $^{-1}$ **(-1.43)*** is approximately ***134.37°*** or ***2.35*** radians.

Problem 6:

Using a calculator, find ***csc** $^{-1}$ **(1.942)*** in degrees and radians rounded to two decimal places.

Since there is no calculator key to evaluate the inverse cosecant, we must use the inverse sine as follows.

Find ***sin** $^{-1}$ **(1/1.942)*** which equals approximately ***30.99°*** or ***0.54*** radians.

This is in the range $-\frac{\pi}{2} < y < \frac{\pi}{2}$ of the inverse cosecant function!

Therefore, ***csc** $^{-1}$**(1.942)*** is approximately ***30.99°*** or ***0.54*** radians.