



DETAILED SOLUTIONS AND CONCEPTS - INTRODUCTION TO EXPONENTIAL AND LOGARITHMIC EQUATIONS

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PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

The logarithm is perhaps the single, most useful arithmetic concept in all the sciences; and an understanding of them is essential to an understanding of many scientific ideas. Logarithms may be defined and introduced in several different ways. But for our purposes, let's adopt a simple approach.

Logarithms were invented by several mathematicians, who searched for a way to find unknown values of powers in exponential equations. Today they are used to understand phenomena such as loudness of sound, earthquake magnitude, an object's cooling time, etc.

Definition of the Logarithm

An exponential equation of the form $M = b^a$ can be written in logarithmic form as $\log_b M = a$ (pronounced *log base b of M*).

b MUST be a positive number other than 1 and a any real number. Therefore, M is NEVER equal to 0 and always positive.

NOTE: A negative M can occur, but only if a is an imaginary number. This concept is discussed in a course on Complex Analysis.

Problems:

The exponential equation $32 = 2^5$ can be written as the logarithmic equation $\log_2 32 = 5$ and vice versa!

Please note that $\log_2 32$ is asking "To what power must we raise 2 to get 32?"

The exponential equation $1000 = 10^3$ can be written as the logarithmic equation $\log_{10} 1000 = 3$.

The exponential and logarithm bases that occur most frequently in application problems are **e** and **10**, where **e** is the famous irrational number **2.718281828459045...**, often rounded to **2.72**.

Point of Interest:

The number **e** was found while investigating what happens to the value of $(1 + \frac{1}{m})^m$ as **m** gets infinitely large.

See for yourself by using increasingly larger values of **m**. The value of $(1 + \frac{1}{m})^m$ will never become larger than **2.718**.

All calculators contain the value of **e**. You MUST use it and NOT **2.718** !!!

Calculator Tip for the TI 30X II B or TI 30X II S calculator: The keystrokes 2nd LN allow you to access the value of e.

Warning: Left parenthesis will open when you activate the calculator function e^x , specifically you will see $e^x ($. After you type in the power, you MUST type in the right parenthesis, namely $)$, before you press ENTER.

For Problem, if you want to evaluate the exponential expression e^3 type the following

2nd	LN	3)	Enter
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If you just want to see the value of the number **e**, type the following into your calculator

2nd	LN	1)	Enter
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All calculators also have a **LOG-key** (base 10) and an **LN-key** (base e). To evaluate other bases you must use the *Change of Base Property*.

Calculator Tip for the TI 30X II B or TI 30X II S calculator:

To evaluate logarithms, we MUST have a calculator. Trying to evaluate logarithms by hand is not only extremely cumbersome, but also waaaaay beyond the scope of this lecture. In a calculus course it is often shown how to find the values of logarithms by hand and how the calculator is programmed to find values of logarithms. However, not even then do we find these values by hand.

$$\ln 2 \approx 0.6931$$

LN	2)	Enter
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$$\log 1000 = 3$$

LOG	1000)	Enter
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Warning: Left parenthesis will open when you activate *LN* or *LOG* specifically you will see *ln* (or *log* (. After you press LN or LOG, you **MUST** type in the right parenthesis, namely ")", before you press ENTER.

Standard Notations:

$\log_e x = \ln x$ Logarithms to **base e** are called **natural logarithms**.

The **e** is left off and **log** changes to **ln**. **Note that the first letter is a lower case L and NOT a capital i !!**

$\log_{10} x = \log x$ Logarithms to **base 10** are called **common logarithms**.

The **10** is left off.

Change of Base Property

Given two logarithm bases, say **a** and **b**, we can state the following:

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Problem 1:

Change $4^x = 2.5$ to an equivalent logarithmic equation.

That is, $\log_4 2.5 = x$.

Problem 2:

Change $2^5 = 32$ to an equivalent logarithmic equation.

That is, $\log_2 32 = 5$.

Problem 3:

Change $\log_2 8 = 3$ to an equivalent equation involving an exponent.

That is, $8 = 2^3$.

Problem 4:

Change $\log_x 2 = 4$ to an equivalent exponential equation.

That is, $2 = x^4$.

Problem 5:

Change $\log_6 5 = x$ to an equivalent exponential equation.

That is, $5 = 6^x$.

Problem 6:

Evaluate $\log_4 21$ using a calculator. Round to 4 decimal places.

In order to use the calculator, we have to use the *Change of Base Property* to change the logarithm base to either **base 10** or **base e**.

Method 1:

Let's use **base 10**.

Given $\log_b M = \frac{\log_a M}{\log_a b}$, we have $a = 10$ and $b = 4$

then

$$\log_4 21 = \frac{\log 21}{\log 4} \approx 2.1962$$

Method 2:

Let's use **base e**.

We will use **$a = e$** and **$b = 4$**

then $\log_4 21 = \frac{\ln 21}{\ln 4} \approx 2.1962$

Please note that $\log_4 21$ is asking "To what power must we raise 4 to get 21?" We have just found this power to be approximately 2.1962 !!!