DETAILED SOLUTIONS AND CONCEPTS - SOLVING EXPONENTIAL EQUATIONS

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PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

The following are some examples of the types of exponential equations that we are going to solve in this section.

$$10^x = 5.71 \qquad 2^{x+2} = 3^{2x+1} \qquad 7e^x = 15 \qquad 9^{2x} = 27^{x+1}$$

Strategy for Solving Exponential Equations

- If necessary, isolate the exponential expression on one side of the equation with a coefficient of ***1***.
- By definition, if ***M = N***, then $\log_b M = \log_b N$. Expressed differently, either

  place the word ***log*** (***base 10*** is assumed) in front of the entire right side and the entire left side of the exponential equation

  or

  place the word ***ln*** (***base e*** is assumed) in front of the entire right side and the entire left side of the exponential equation.

- Your exponential equation is now a logarithmic equation. With the help of the *Power Rule* you can now "free" the variable from its exponential position.
- Solve for the variable.

Problem 1:

Solve $10^x = 5.71$. Round to 4 decimal places.

Method 1:

Since the exponential expression is already isolated we'll place the word ***ln*** (log base e) in front of the right side and in front of the left side

$$\ln 10^x = \ln 5.71$$

Using the *Power Rule*, we get

$$x \ln 10 = \ln 5.71$$

Please note that by the basic *Logarithm Properties*

$$\ln 10 = \ln_e 10 \neq 1$$

Therefore, we have to divide both sides by $\ln 10$

$x = \dfrac{\ln 5.71}{\ln 10}$ and using the calculator, we find that $x \approx .7566$.

Method 2:

Let's do this problem again. This time we'll use the common logarithm, that is log base 10.

$$10^x = 5.71$$

Now, place the word $\log$ (log base 10) in front of the right side and in front of the left side

$$\log 10^x = \log 5.71$$

$$x \log 10 = \log 5.71$$

Please note that by the basic *Logarithm Properties*

$$\log 10 = \log_{10} 10 = 1$$

So that we can write $x = \log 5.71$ and using the calculator, we find that $x \approx .7566$.

As you can see, we get the same solution no matter which logarithm base we used. However, Method 2 was a little faster because the base of the logarithm matched the base of the exponential expression.

Problem 2:

Solve $7e^x = 15$. Round to 4 decimal places.

Method 1:

Let's use the natural logarithm, that is log base e.

Isolate the exponential expression

$$e^x = \frac{15}{7}$$

$$\ln e^{x} = \ln \tfrac{15}{7}$$

Using the *Power Rule*, we get

$$x \ln e = \ln \tfrac{15}{7}$$

Please note that by the basic *Logarithm Properties*
$\ln e = \log_e e = 1$

So that we can write $x = \ln \tfrac{15}{7}$ and using the calculator, we find that
$x \approx .7621$

Method 2:

This time we'll use the common logarithm, that is log base 10.

Isolate the exponential expression

$$e^{x} = \tfrac{15}{7}$$

$$\log e^{x} = \log \tfrac{15}{7}$$

$$x \log e = \log \tfrac{15}{7}$$

Please note that $\log e = \log_{10} e \neq 1$!

Therefore, we have to divide both sides by $\log e$

$x = \dfrac{\log \frac{15}{7}}{\log e}$ and using the calculator, we find again that $x \approx .7621$.

As you can see, we get the same solution no matter which logarithm base we used. However, in this case Method 1 was a little faster than Method 2 because the base of the logarithm matched the base of the exponential expression.

Method 3:

This time, let's pretend that we forgot to isolate the exponential expression.

Then $\ln 7e^{x} = \ln 15$

Please note that $x \ln 7e \neq \ln 15$ because the power x only affects the number e and NOT the number 7 ! ! ! ! !

Instead, you MUST use the *Product Rule* to solve as follows

$$\ln 7 + \ln e^{x} = \ln 15$$

$$\ln 7 + x \ln e = \ln 15$$

$$x = \ln 15 - \ln 7$$

and using the calculator, we find again that $x \approx .7621$.

Problem 3:

Solve $16^{x-1} = \frac{1}{2}$.

Method 1:

Let's use the natural logarithm in the solution process!

$$\ln 16^{x-1} = \ln \tfrac{1}{2}$$

$(x-1)\ln 16 = \ln \frac{1}{2}$. Notice the parentheses around ***(x - 1)***!!!

$$x - 1 = \frac{\ln \frac{1}{2}}{\ln 16}$$

$$x - 1 = -.25$$

$$x = .75$$

Method 2:

Please be aware that we can only use this method because the numbers 16 and $\frac{1}{2}$ have the same base when written in exponential form. That is, $\frac{1}{2} = 2^{-1}$ and $16 = 2^4$.

Therefore, let's rewrite $\ln 16^{x-1} = \ln \frac{1}{2}$ as $\left(2^4\right)^{x-1} = 2^{-1}$

and $2^{4(x-1)} = 2^{-1}$

Since the two expressions are obviously equal, and the bases are also obviously equal, then the two exponents also MUST be equal to each other.

Therefore, $4(x-1) = -1$

$$4x - 4 = -1$$

$4x = 3$ and $x = \frac{3}{4} = .75$

Problem 4:

Solve $9^{2x} = 27^{x+1}$ not using logarithms!

Let's solve this exponential equation using the fact that 9 and 27 have the same base! That is,

$$(3^2)^{2x} = (3^3)^{x+1}$$
$$3^{4x} = 3^{3(x+1)}$$
$$4x = 3x + 3$$
$$x = 3$$

Problem 5:

Solve $5^{x-2} = 3^{2x+1}$. Round to 3 decimal places.

Let's use the common logarithm in the solution process!

$$\log 5^{x-2} = \log 3^{2x+1}$$

Using the *Power Rule* we find

$$(x-2)\log 5 = (2x+1)\log 3$$

Next, we distribute the logarithmic expressions

$$x\log 5 - 2\log 5 = 2x\log 3 + \log 3$$

and collect the expressions containing the variable on one side

$$x\log 5 - 2x\log 3 = \log 3 + 2\log 5$$

this allows us to factor out the variable and isolate it as follows

$$x(\log 5 - 2\log 3) = \log 3 + 2\log 5$$
$$x = \frac{\log 3 + 2\log 5}{\log 5 - 2\log 3} \approx -7.345$$

Problem 6:

How many years will it take for an initial investment of ***$10,000*** to grow to ***$25,000***? Assume a rate of interest of ***2.5%*** compounded continuously. Round your answer to a whole number. Use the formula $A = Pe^{rt}$, where P is the initial investment, A is the accumulated amount, t is the time in years and r is the interest rate in decimals.

NOTE: Do not round until you find the final answer!

$$25000 = 10000\, e^{0.025t}$$

$$2.5 = e^{0.025t}$$

$$\ln 2.5 = \ln e^{0.025t}$$

$$\ln 2.5 = 0.025t \ln e$$

$$t = \frac{\ln 2.5}{0.025} \approx 37$$

It takes *approximately 37 years* for $10,000 to grow to $25,000 at a rate of interest of 2.5%.

Problem 7:

The number of bacteria A in a certain culture is given by the growth model $A = 250e^{kt}$. Find the growth constant k knowing that $A = 280$ when $t = 5$. Round your answer to four decimal places.

NOTE: Do not round until you find the final answer!

$$280 = 250e^{k(5)}$$

$$1.12 = e^{5k}$$

$$\ln 1.12 = \ln e^{5k}$$

$$\ln 1.12 = 5k \ln e$$

$$k = \frac{\ln 1.12}{5} \approx 0.0227$$

The growth constant k equals approximately *0.0227*.

Problem 8:

The half-life of a radioactive substance is ***950 years***.. Find the constant k rounded to seven decimal places. Do not use scientific notation! Hint: Half-life means that exactly one-half of the original amount or size of the substance is left after a certain number of years of growth/decay. Use the *Exponential Growth/Decay Model* $A = A_0 e^{kt}$, where A_0 is the original amount, A is the accumulated amount, t is the time in years and k is the growth constant.

We know that after 950 years one-half of the original amount $\left(\frac{1}{2} A_0\right)$ is left. Therefore,

$$\frac{1}{2} A_0 = A_0 e^{k(950)}$$

Then,

$$\frac{1}{2} = e^{k(950)}$$

$$\ln \frac{1}{2} = \ln e^{950k}$$

$$\ln \frac{1}{2} = 950k \ln e$$

$$k = \frac{\ln \frac{1}{2}}{950} \approx -7.296 \times 10^{-4} = -0.0007296$$

The decay constant k equals approximately - *0.0007296*.

Problem 9:

The next problem involves carbon-14 dating which is used to determine the age of fossils and artifacts. The method is based on considering the percentage of a half-life of carbon-14 of approximately 5715 years. Specifically, the model for carbon-14 is

$$A = A_0 e^{-0.000121t}.$$

In 1947, an Arab Bedouin herdsman found earthenware jars containing what are known as the Dead Sea scrolls. Analysis at that time indicated that the scroll wrappings contained 76% of their original carbon-14. Estimate the age of the scrolls in 1947. Round your answer to a whole number.

We know that A, the amount present is 76% of the original amount A_0. Therefore, we can use the model to write

$$0.76 A_0 = A_0 e^{-0.000121t}$$

$$0.76 = e^{-0.000121t}$$

$$\ln 0.76 = \ln e^{-0.000121\,t}$$

$$\ln 0.76 = -0.000121\,t \ln e$$

$$t = \frac{\ln 0.76}{-0.000121} \approx 2268$$

The Dead Sea Scrolls were *approximately 2268 years* old in 1947.