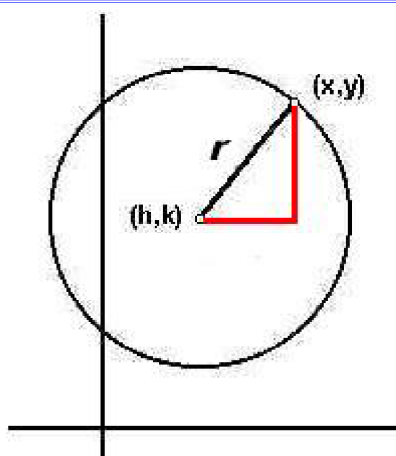




DETAILED SOLUTIONS AND CONCEPTS - CIRCLES AND SEMICIRCLES
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 Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!



Standard Form of the Equation of a Circle

This equation is derived by using the Pythagorean Theorem on the right triangle as shown in the picture. Thus, the equation of the circle with the center at (h, k) and radius of length r is

$$(x - h)^2 + (y - k)^2 = r^2$$

You recognize that an equation describes a circle by the following:

- there is always an x and a y to the second power
- the x and y to the second power must have the same sign when on the same side of the equation
- the x and y to the second power always have the same coefficient
- there could be an x and/or a y to the first power (means that the center is not at the origin) as well as constants

The Equations that describe Semicircles

$$(y - k)^2 = r^2 - (x - h)^2$$

$$y - k = \pm\sqrt{r^2 - (x - h)^2}$$

$$y = \pm\sqrt{r^2 - (x - h)^2} + k$$

$$(x - h)^2 = r^2 - (y - k)^2$$

$$x - h = \pm\sqrt{r^2 - (y - k)^2}$$

$$x = \pm\sqrt{r^2 - (y - k)^2} + h$$

Semicircle - Upper Half of the Circle

$$y = \sqrt{r^2 - (x - h)^2} + k$$

Semicircle - Lower Half of the Circle

$$y = -\sqrt{r^2 - (x - h)^2} + k$$

Semicircle - Right Half of the Circle

$$x = \sqrt{r^2 - (y - k)^2} + h$$

Semicircle - Left Half of the Circle

$$x = -\sqrt{r^2 - (y - k)^2} + h$$

Special Case of the Semicircle:

A circle with the standard equation of $x^2 + y^2 = r^2$, where r is the radius, has its center at the origin $(0, 0)$.

Problem 1:

Given the equation of a circle $(x + 2)^2 + (y - 7)^2 = 9$, find the coordinates of its center and its radius.

You must first convert the equation to standard form. That is, the minus sign after the x and the y must be shown to state the center of this circle.

$$[x - (-2)]^2 + (y - 7)^2 = 9$$

The coordinates of the center of this circle are $(-2, 7)$ and the radius is 3 .

Problem 2:

Given the equation of a circle $x^2 + y^2 + 4x - 6y - 23 = 0$, find the coordinates of its center and its radius.

To find this information you have to complete the square on both x and y .

First we will group all x-terms and all y-terms. We will also bring the constant to the other side:

$$(x^2 + 4x \quad) + (y^2 - 6y \quad) = 23$$

Next, we will complete the square for the x-terms and the y-terms.

$$[x^2 + 4x + (\frac{4}{2})^2] + [y^2 - 6y + (\frac{-6}{2})^2] = 23 + (\frac{4}{2})^2 + (\frac{-6}{2})^2$$

Note that since we added $\left(\frac{4}{2}\right)^2 = 4$ and $\left(\frac{-6}{2}\right)^2 = 9$ to the left side of the equation, we must also add both to the right side. Otherwise the original equality would be changed.

Simplifying, we get

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = 23 + 4 + 9$$

and factoring and combining like terms on the right produces

$$(x + 2)^2 + (y - 3)^2 = 36$$

The coordinates of the center of the circle are $(-2, 3)$ and the radius is 6 .

Problem 3:

Given the equation of a circle $x^2 + y^2 = 36$

- find the coordinates of its center and its radius
- find the coordinates of the x- and y-intercepts

Center and Radius:

Written in standard form $(x - 0)^2 + (y - 0)^2 = 36$, we can clearly see that this circle has its center at $(0, 0)$ and a radius of 6 .

Coordinates of the x- and y-intercepts:

- Coordinates of the x-intercepts (when $y = 0$):

$$x^2 + 0^2 = 36$$

$$x^2 = 36$$

$$x = \pm 6$$

The coordinates of the x-intercepts are $(-6, 0)$ and $(6, 0)$.

- Coordinates of the y-intercepts (when $x = 0$):

$$0^2 + y^2 = 36$$

$$y^2 = 36$$

$$y = \pm 6$$

The coordinates of the y-intercepts are $(0, -6)$ and $(0, 6)$.

Problem 4:

Given the equation of a circle $x^2 + y^2 = 9$

- find the coordinates of its center and its radius
- find the coordinates of the x- and y-intercepts

Center and Radius:

Written in standard form $(x - 0)^2 + (y - 0)^2 = 9$, we can clearly see that this circle has its center at $(0, 0)$ and a radius of 3 .

Coordinates of the x- and y-intercepts:

- Coordinates of the x-intercepts (when $y = 0$):

$$x^2 + 0^2 = 9$$

$$x^2 = 9$$

$$x = \pm 3$$

The coordinates of the x-intercepts are $(-3, 0)$ and $(3, 0)$.

- Coordinates of the y-intercepts (when $x = 0$):

$$0^2 + y^2 = 9$$

$$y^2 = 9$$

$$y = \pm 3$$

The coordinates of the y-intercepts are $(0, -3)$ and $(0, 3)$.

Problem 5:

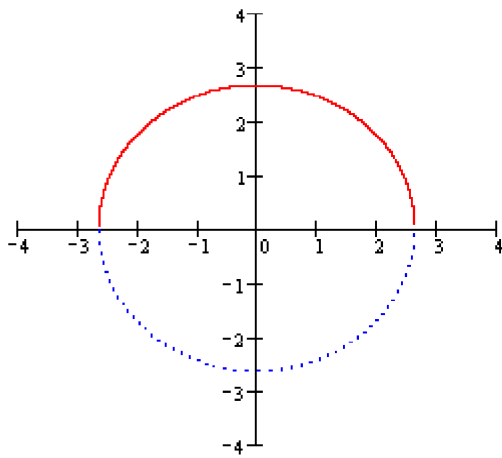
Find the equations for the upper and lower half of the circle $x^2 + y^2 = 7$.

$$y^2 = 7 - x^2$$

$$y = \pm\sqrt{7 - x^2}$$

Equation of the upper half of the circle is $y = \sqrt{7 - x^2}$

Equation of the lower half of the circle is $y = -\sqrt{7 - x^2}$



Problem 6:

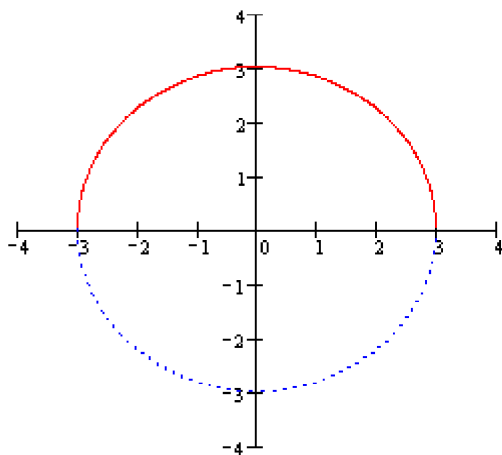
Find the equations for the upper, lower, right, and left half of the circle $x^2 + y^2 = 9$.

$$y^2 = 9 - x^2 \quad x^2 = 9 - y^2$$

$$y = \pm\sqrt{9 - x^2} \quad x = \pm\sqrt{9 - y^2}$$

Equation of the upper half of the circle is $y = \sqrt{9 - x^2}$

Equation of the lower half of the circle is $y = -\sqrt{9 - x^2}$



Equation of the right half of the circle is $x = \sqrt{9 - y^2}$

Equation of the left half of the circle is $x = -\sqrt{9 - y^2}$

