$$\lim_{x \to \infty} \int_2^3 \frac{1}{dx} dy$$

THE PRODUCT AND THE QUOTIENT RULE

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You can find the proofs of these rules in the online textbook as a separate document.

Product Rule:

Let u and v be a function of x.

If $f(x) = uv$ then $f'(x) = u'v + uv'$

Quotient Rule:

Let u and v be a function of x.

If $f(x) = \frac{u}{v}$ then $f'(x) = \frac{u'v - uv'}{v^2}$

Problem 1:

For $f(x) = (x^3 - 2)(2x + 1)$, find $f'(x)$

Since we are dealing with a product, we could use the *Product Rule* or we could simplify the product and use the *Basic Differentiation Rules* discussed in the previous unit.

Method 1 - Use the *Basic Differentiation Rules (Simple Power Rule with Constant Multiple Rule)*

$f(x) = (x^3 - 2)(2x + 1) = 2x^4 + x^3 - 4x - 2$, then
$f'(x) = 8x^3 + 3x^2 - 4$

Method 2 - Use the *Product Rule*

We know that $f'(x) = u'v + uv'$. Let's find all of the parts we need for this derivative.

That is, $u = x^3 - 2$, and $v = 2x + 1$,

then $u' = 3x^2$ and $v' = 2$

$$f'(x) = 3x^2(2x + 1) + (x^3 - 2)(2) = 6x^3 + 3x^2 + 2x^3 - 4$$

and $f'(x) = 8x^3 + 3x^2 - 4$

Problem 2:

For $f(x) = 4(x^3 - 2x + 1)$, find $f'(x)$

Since we are dealing with a product, we could use the *Product Rule* or we could simplify the product and use the *Basic Differentiation Rules* discussed in the previous unit.

Method 1 - Use the *Basic Differentiation Rules (Simple Power Rule with Constant Multiple Rule)*

$f(x) = 4(x^3 - 2x + 1) = 4x^3 - 8x + 4$, then $f'(x) = 12x^2 - 8$

Method 2 - Use the *Basic Differentiation Rules (Constant Multiple Rule and Simple Power Rule with Constant Multiple Rule)*

$f(x) = 4(x^3 - 2x + 1)$, then $f'(x) = 4(3x^2 - 2) = 12x^2 - 8$

Method 3 - Use the *Product Rule*

We know that $f'(x) = u'v + uv'$. Let's find all of the parts we need for this derivative.

That is, $u = 4$, and $v = x^3 - 2x + 1$,

then $u' = 0$ and $v' = 3x^2 - 2$

$f'(x) = 0(x^3 - 2x + 1) + 4(3x^2 - 2)$

and $f'(x) = 12x^2 - 8$

Problem 3:

Differentiate $f(x) = \dfrac{3x^2 - 7x + 2}{x}$. Write the derivative as ONE SINGLE fraction.

Since we are dealing with a fraction, we could use the *Quotient Rule* or we could simplify the fraction and use the *Basic Differentiation Rules* discussed in the previous unit. However, we could also use the *Product Rule*!

Method 1 - Use the *Basic Differentiation Rules (Simple Power Rule with Constant Multiple Rule)*

$f(x) = \dfrac{3x^2 - 7x + 2}{x} = 3x - 7 + 2x^{-1}$, then $f'(x) = 3 - 2x^{-2} = 3 - \dfrac{2}{x^2}$

Written as ONE SINGLE fraction, we get $f'(x)=\frac{3x^2-2}{x^2}$.

NOTE: Whenever possible you should ALWAYS write your solutions without negative exponents! Also, it is often desirable to change solutions to ONE SINGLE fraction, if possible!

Method 2 - Use the *Quotient Rule*

We know that $f'(x)=\frac{u'v-uv'}{v^2}$. Let's find all of the parts we need for this derivative.

That is $u=3x^2-7x+2$, and $v=x$,

then $u'=6x-7$ and $v'=1$

$$f'(x)=\frac{(6x-7)(x)-(3x^2-7x+2)(1)}{x^2}$$
$$=\frac{6x^2-7x-3x^2+7x-2}{x^2}$$

and $f'(x)=\frac{3x^2-2}{x^2}$

NOTE: Whenever possible you should ALWAYS combine like terms in your solutions!

Method 3 - Use the *Product Rule*

$$f(x)=\frac{3x^2-7x+2}{x}=(3x^2-7x+2)x^{-1}$$

We know that $f'(x)=u'v+uv'$. Let's find all of the parts we need for this derivative.

That is, $u=3x^2-7x+2$, and $v=x^{-1}$,

then $u'=6x-7$ and $v'=-x^{-2}$

$$f'(x)=(6x-7)x^{-1}+(3x^2-7x+2)(-x^{-2})$$

$$f'(x)=6-7x^{-1}-3+7x^{-1}-2x^{-2}=6-\frac{7}{x}-3+\frac{7}{x}-\frac{2}{x^2}=3-\frac{2}{x^2}$$

When we write the derivative as ONE SINGLE fraction we get

$$f'(x) = \frac{3x^2 - 2}{x^2}$$ just like in Methods 1 and 2 above.

Problem 4:

Find the derivative of $f(x) = \frac{4x^3 + 5x - 9}{2}$ written as ONE SINGLE fraction.

Since we are dealing with a fraction, we could use the *Quotient Rule* or we could simplify the fraction and use the *Basic Differentiation Rules* discussed in the previous unit. Using the *Product Rule* for this example should not be considered!

Method 1 - Use the *Basic Differentiation Rules - Simple Power Rule with Constant Multiple Rule*

$$f(x) = \frac{4x^3 + 5x - 9}{2} = 2x^3 + \tfrac{5}{2}x - \tfrac{9}{2}$$, then $f'(x) = 6x^2 + \tfrac{5}{2}$

and written as ONE SINGLE fraction, we get $f'(x) = \frac{12x^2 + 5}{2}$

Method 2 - Use the *Quotient Rule*

We know that $f'(x) = \frac{u'v - uv'}{v^2}$. Let's find all of the parts we need for this derivative.

That is, $u = 4x^3 + 5x - 9$, and $v = 2$,

fhen $u' = 12x^2 + 5$ and $v' = 0$

$$f'(x) = \frac{(12x^2 + 5)(2) - (4x^3 + 5x - 9)(0)}{2^2}$$
$$= \frac{24x^2 + 10}{4}$$

and $f'(x) = \frac{12x^2 + 5}{2}$ just like in Method 1.

Problem 5:

Find the derivative of $g(x)=\frac{2x-x^2}{\sqrt{x}}$ using the *Quotient Rule*. Write your answer without negative exponents.

$$g(x)=\frac{2x-x^2}{\sqrt{x}}=\frac{2x-x^2}{x^{1/2}}$$

Let $u=2x-x^2$ and $v=x^{1/2}$, then $u'=2-2x$ and $v'=\frac{1}{2}x^{-1/2}$.

$$g'(x)=\frac{(2-2x)x^{1/2}-(2x-x^2)(\frac{1}{2}x^{-1/2})}{(x^{1/2})^2}$$

Now, let's write the derivative without negative exponents.

Actually, $g'(x)=\dfrac{(2-2x)x^{1/2}-\dfrac{2x-x^2}{2x^{1/2}}}{x}$, which is a complex fraction that ALWAYS must be simplified.

Intermediate Algebra presented several methods to simplify a complex fraction. The easiest way is to multiply both the numerator and the denominator of the complex fraction by the LCD derived from the numerator and denominator of the complex fraction.

In our case, $2x^{1/2}$ is the denominator in the numerator of the complex fraction and ***1*** is the denominator in the denominator of the complex fraction!

Therefore, the LCD (Least Common Denominator) is $2x^{1/2}$.

Then $g'(x)=\dfrac{(2-2x)x^{1/2}-\dfrac{2x-x^2}{2x^{1/2}}}{x}\cdot\dfrac{2x^{1/2}}{2x^{1/2}}$

Please note that we are actually multiplying

$$g'(x)=\frac{(2-2x)x^{1/2}-(2x-x^2)(\frac{1}{2}x^{-1/2})}{x}$$

by the number *1* in the special form $\dfrac{2x^{1/2}}{2x^{1/2}}$ **!!!**

Next, we'll distribute $2x^{1/2}$ to the two terms of the numerator and to the single term in the denominator of the complex fraction.

that is, $$g'(x) = \frac{(2-2x)x^{1/2}\cdot 2x^{1/2} - \frac{2x-x^2}{2x^{1/2}}\cdot 2x^{1/2}}{x\cdot 2x^{1/2}}$$

Combining like terms we get

$$g'(x) = \frac{2(2-2x)x-(2x-x^2)}{2x^{3/2}} = \frac{4x-4x^2-2x+x^2}{2x^{3/2}}$$

$$g'(x) = \frac{2x-3x^2}{2x^{3/2}}$$

and $$g'(x) = \frac{2-3x}{2x^{1/2}}$$