$\lim_{x \to \infty} \int_2^3 \frac{1}{dx} \, dy$ **INTRODUCTION TO LIMITS**

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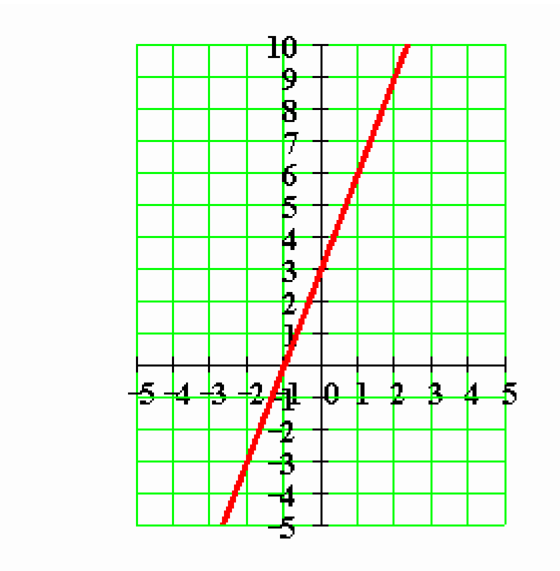
Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

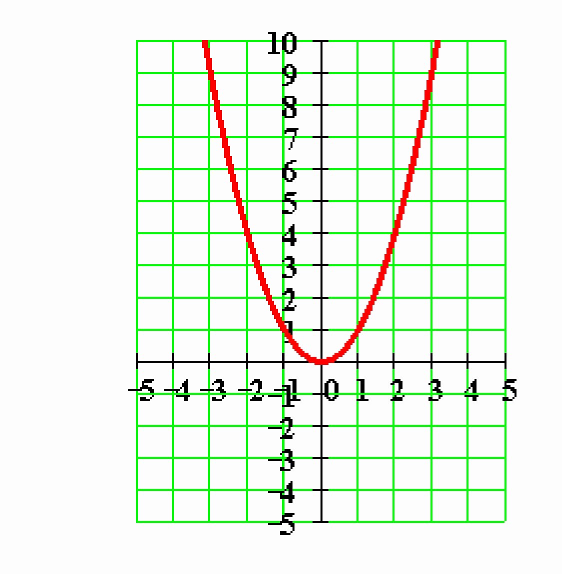
Problem 1:

Given $c = 2$ and $f(x) = 3x + 3$, a linear function with domain $(-\infty, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



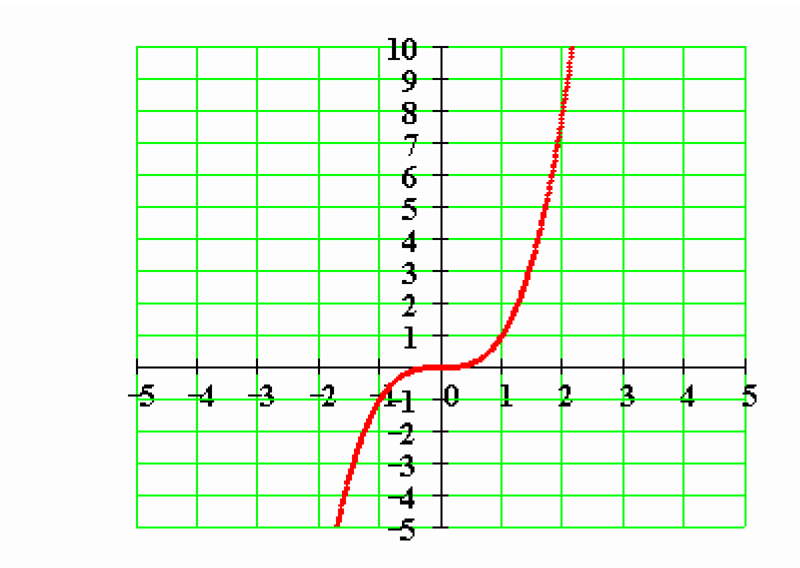
Problem 2:

Given $c = 2$ and $f(x) = x^2$, a quadratic function with domain $(-\infty, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



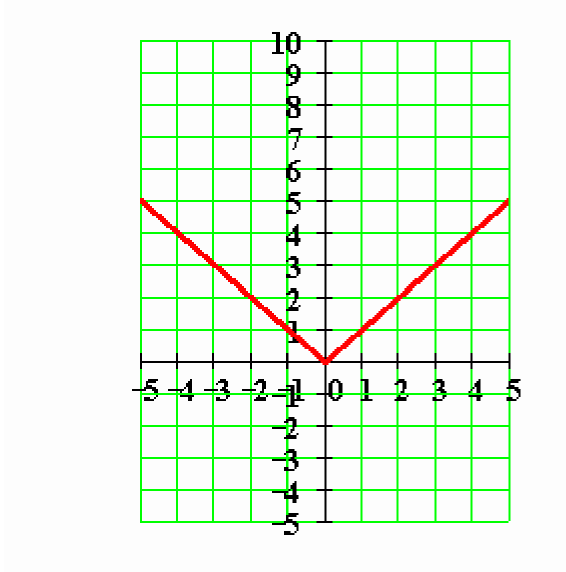
Problem 3:

Given $c = 2$ and $f(x) = x^3$, a cubic function with domain $(-\infty, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



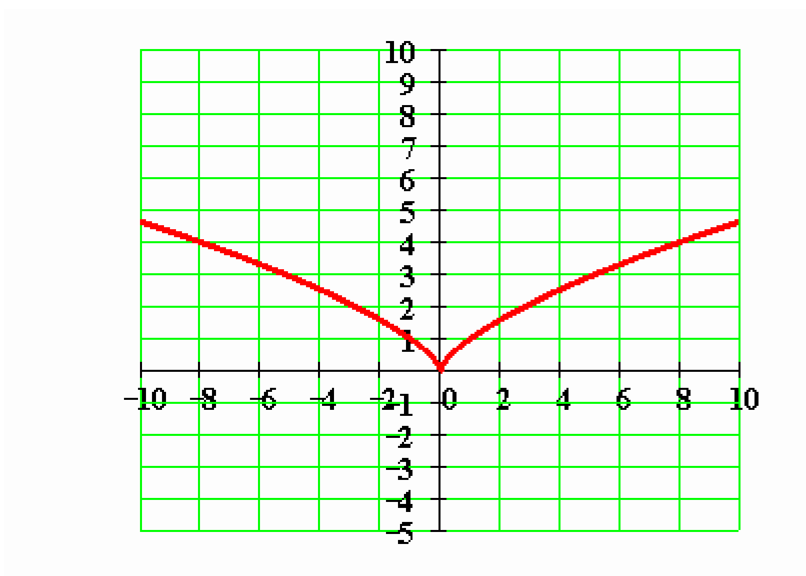
Problem 4:

Given $c = -2$ and $f(x) = |x|$, an absolute-value function with domain $(-\infty, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



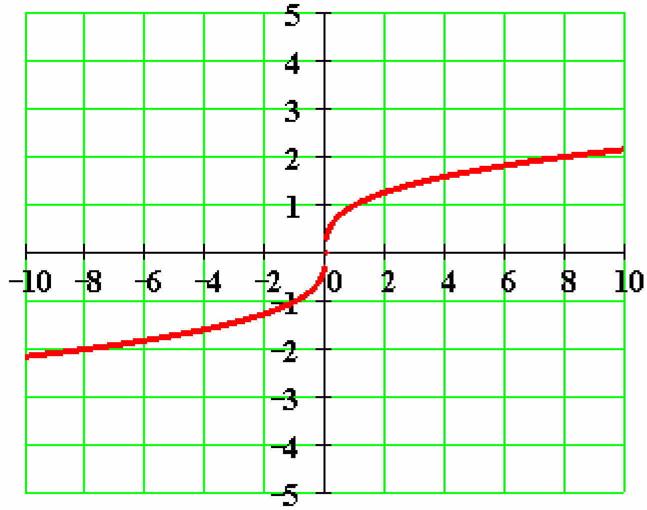
Problem 5:

Given $c = -8$ and $f(x) = x^{2/3}$, a function with domain $(-\infty, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



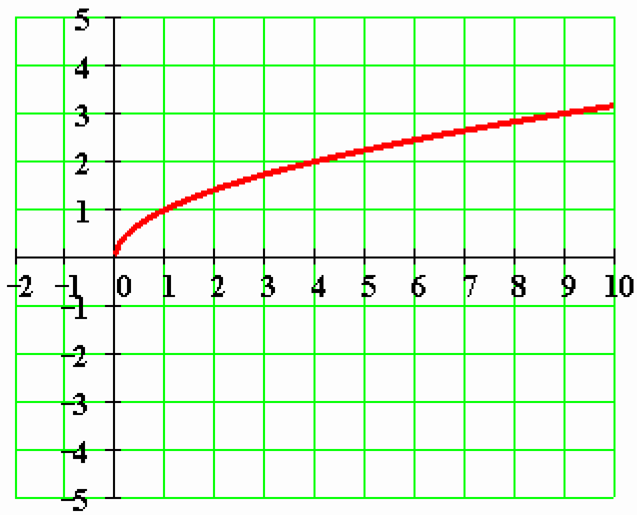
Problem 6:

Given $c = -8$ and $f(x) = x^{1/3}$, a function with domain $(-\infty, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



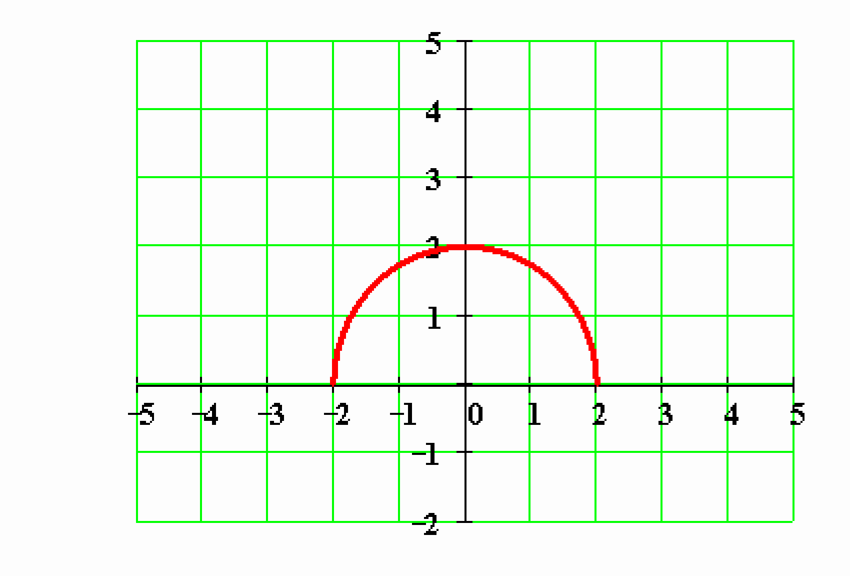
Problem 7:

Given $c = 9$ and $f(x) = \sqrt{x}$, a square-root function with domain $[0, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



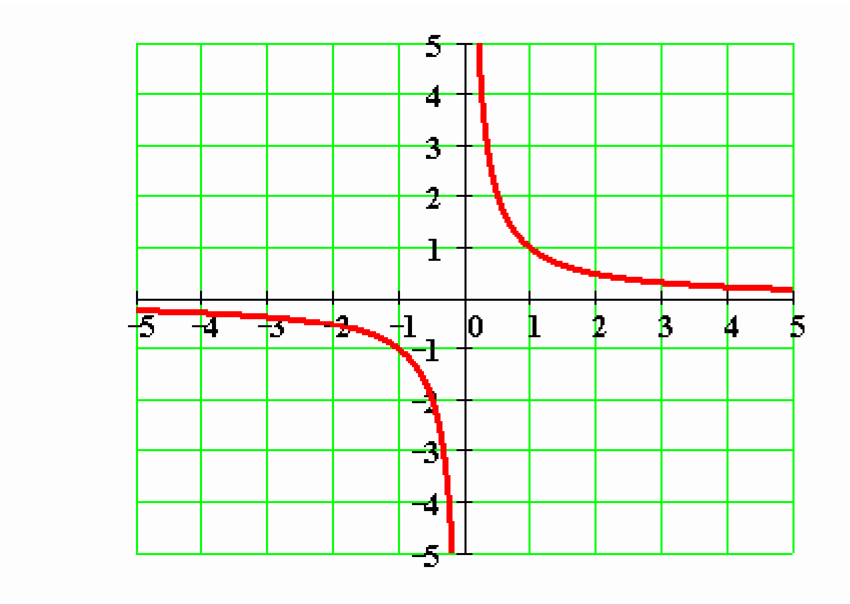
Problem 8:

Given $c = 0$ and $f(x) = \sqrt{4 - x^2}$, a semicircular function with domain $[-2, 2]$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



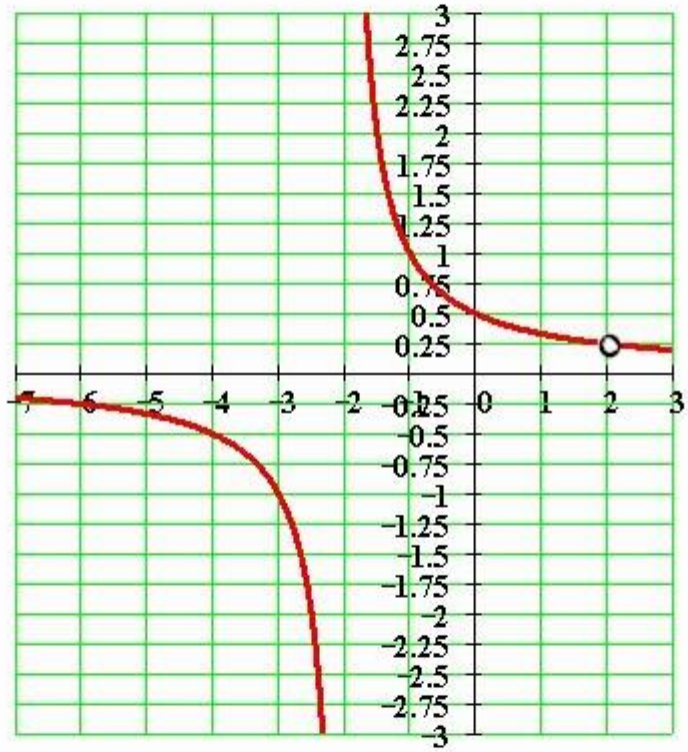
Problem 9:

Given $c = 1$ and $f(x) = \frac{1}{x}$, a rational function (reciprocal function) with domain $\{x \mid x \neq 0\}$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



Problem 10:

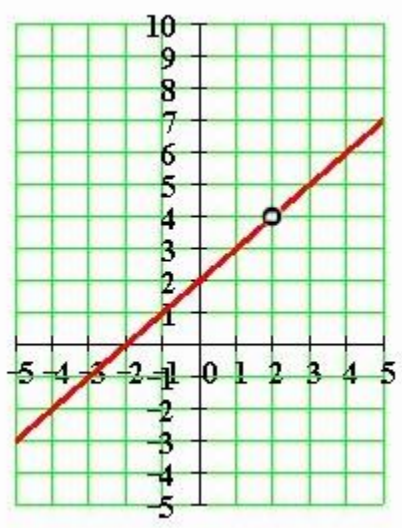
Given $c = -1$ and $f(x) = \frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)}$, a function with domain $\{x \mid x \neq -2, x \neq 2\}$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



Problem 11:

Given $c = 1$ and $f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{x-2}$, a function with domain $\{x \mid x \neq 2\}$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



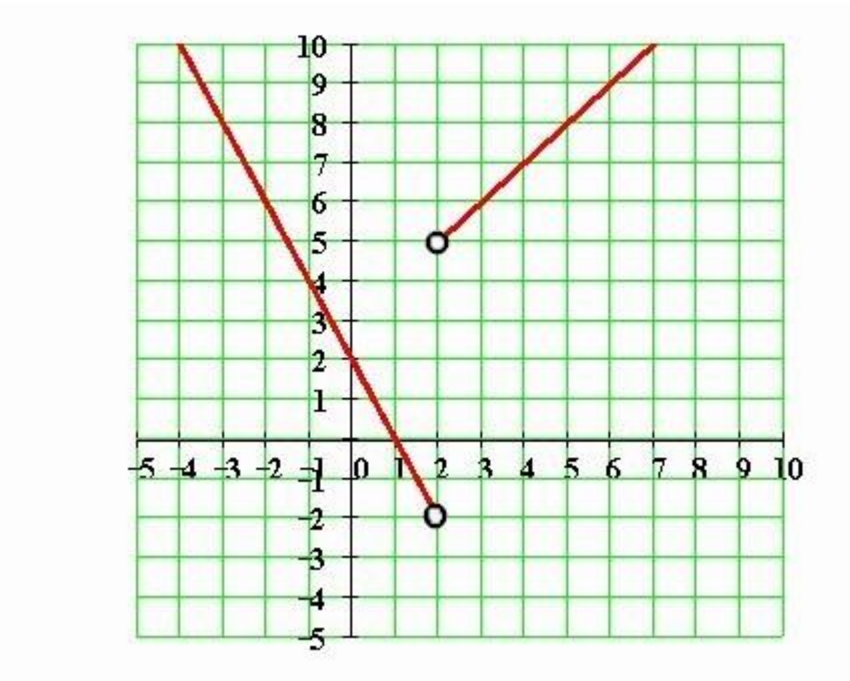
Problem 12:

Given $c = 0$ and $c = 4$ and

$$f(x) = \begin{cases} -2x + 2 & \text{if } x < 2 \\ x + 3 & \text{if } x > 2 \end{cases},$$

a piece-wise defined function with domain $(-\infty, 2) \cup (2, \infty)$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



Problem 13:

Given $c = 0$ and $f(x) = \sqrt{x}$, a square-root function with domain $[0, \infty)$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 7**.

Problem 14:

Given $c = -2$ and $c = 2$ and $f(x) = \sqrt{4 - x^2}$, a semicircular function with domain $[-2, 2]$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 8**.

Problem 15:

Given $c = 0$ and $f(x) = \frac{1}{x}$, a rational function (reciprocal function) with domain $\{x \mid x \neq 0\}$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 9**.

Problem 16:

Given $c = -2$ and $f(x) = \frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)}$, a function with domain $\{x \mid x \neq -2, x \neq 2\}$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 10**.

Problem 17:

Given $c = 2$ and $f(x) = \frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)}$, a function with domain $\{x \mid x \neq -2, x \neq 2\}$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 10**.

Problem 18:

Given $c = 2$ and $f(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2}$, a function with domain $\{x \mid x \neq 2\}$. Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 11**.

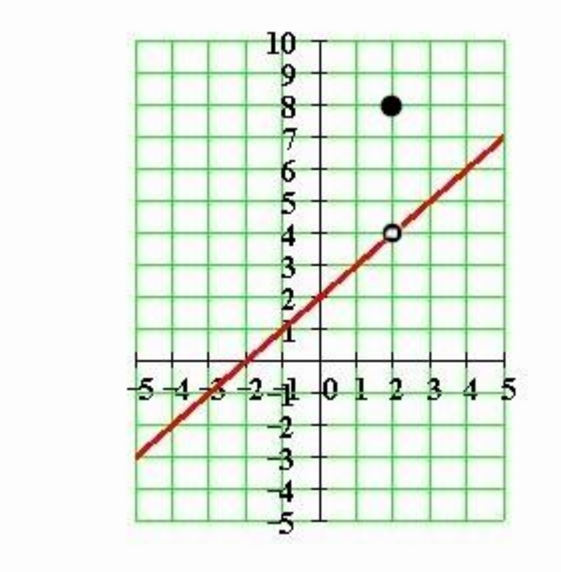
Problem 19:

Given $c = 2$ and

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} & \text{if } x \neq 2 \\ 8 & \text{if } x = 2 \end{cases},$$

a piece-wise defined function with domain $(-\infty, \infty)$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



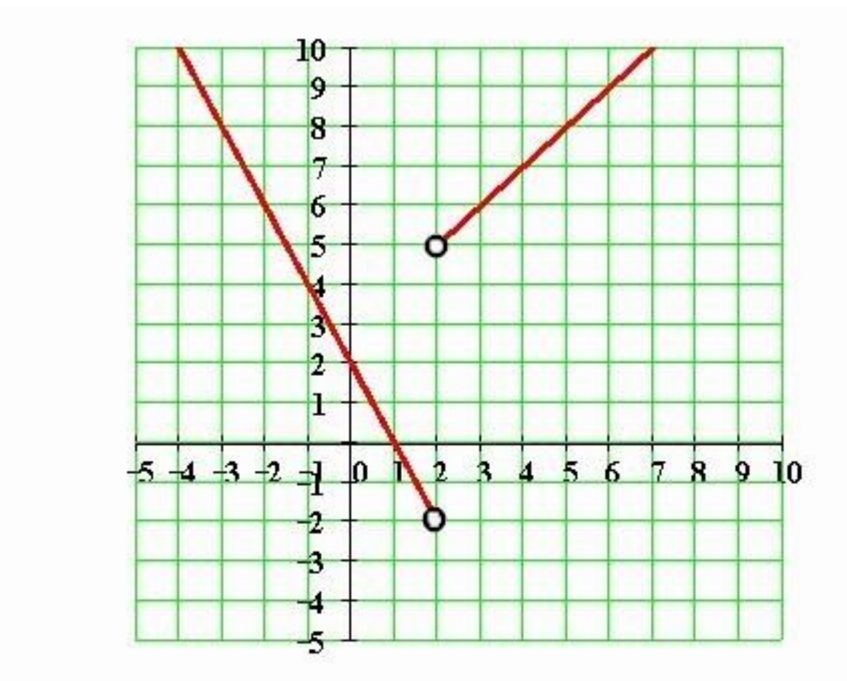
Problem 20:

Given $c = 2$ and

$$f(x) = \begin{cases} -2x + 2 & \text{if } x < 2 \\ x + 3 & \text{if } x > 2 \end{cases},$$

a piece-wise defined function with domain $(-\infty, 2) \cup (2, \infty)$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph in **Problem 12**.



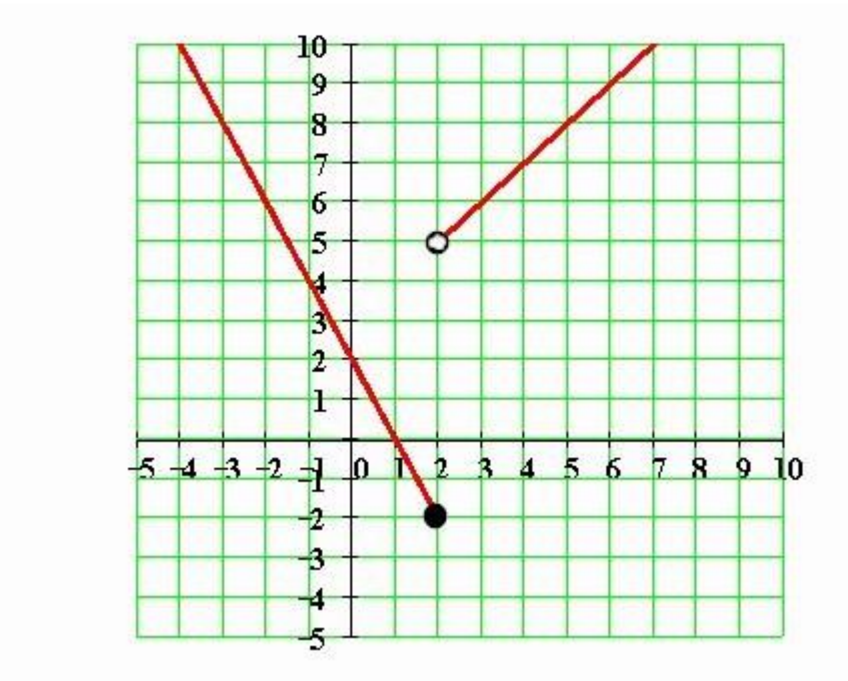
Problem 21:

Given $c = 2$ and

$$f(x) = \begin{cases} -2x + 2 & \text{if } x \le 2 \\ x + 3 & \text{if } x > 2 \end{cases},$$

a piece-wise defined function with domain $(-\infty, \infty)$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



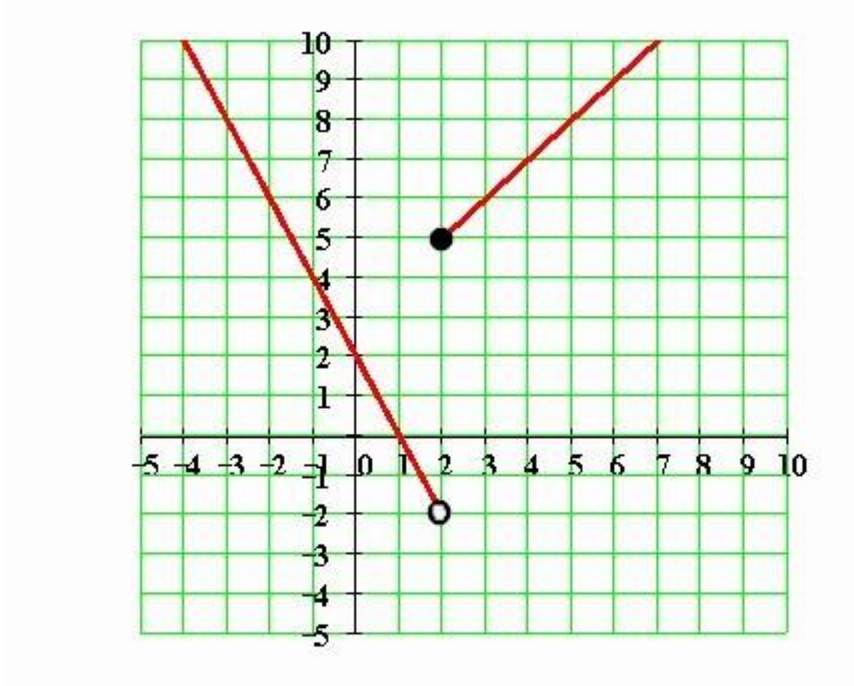
Problem 22:

Given $c = 2$ and

$$f(x) = \begin{cases} -2x + 2 & \text{if } x < 2 \\ x + 3 & \text{if } x \ge 2 \end{cases},$$

a piece-wise defined function with domain $(-\infty, \infty)$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



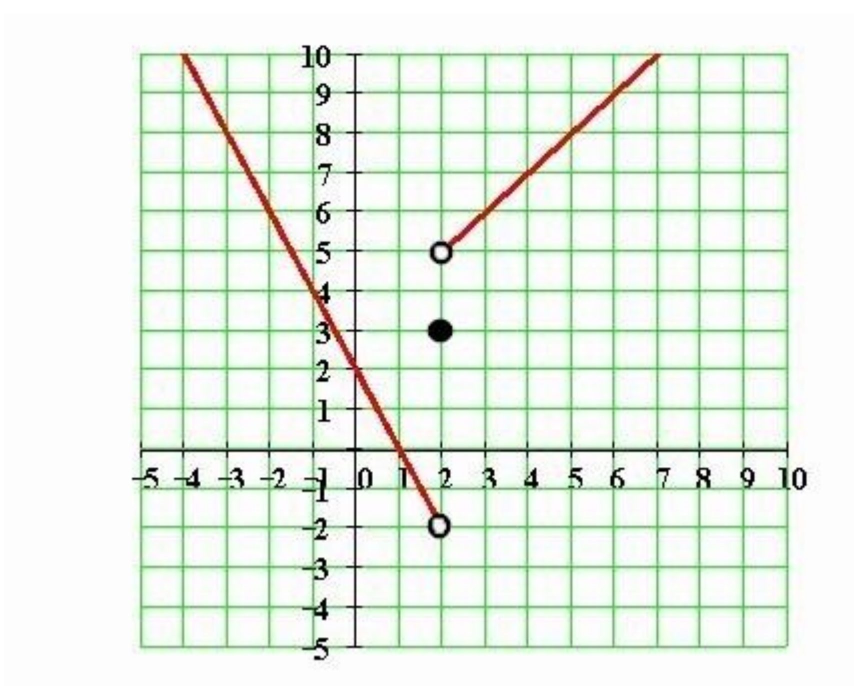
Problem 23:

Given $c = 2$ and

$$f(x) = \begin{cases} -2x + 2 & \text{if } x < 2 \\ x + 3 & \text{if } x > 2 \\ 3 & \text{if } x = 2 \end{cases},$$

a piece-wise defined function with domain $(-\infty, \infty)$.

Calculate $f(c)$ and find $\lim_{x \to c^+} f(x)$, $\lim_{x \to c^-} f(x)$, $\lim_{x \to c} f(x)$ from the graph.



SOLUTIONS

You can find detailed solutions below the link for this problem set!

1. $f(2) = 3(2) + 3 = 9$

$\lim_{x \to 2^+} f(x) = 9$ $\lim_{x \to 2^-} f(x) = 9$ $\lim_{x \to 2} f(x) = 9$

2. $f(2) = (2)^2 = 4$

$\lim_{x \to 2^+} f(x) = 4$ $\lim_{x \to 2^-} f(x) = 4$ $\lim_{x \to 2} f(x) = 4$

3. $f(2) = (2)^3 = 8$

$\lim_{x \to 2^+} f(x) = 8$ $\lim_{x \to 2^-} f(x) = 8$ $\lim_{x \to 2} f(x) = 8$

4. $f(-2) = |-2| = 2$

$\lim_{x \to -2^+} f(x) = 2$ $\lim_{x \to -2^-} f(x) = 2$ $\lim_{x \to -2} f(x) = 2$

5. $f(-8) = (-8)^{2/3} = 4$

$\lim_{x \to -8^+} f(x) = 4$ $\lim_{x \to -8^-} f(x) = 4$ $\lim_{x \to -8} f(x) = 4$

6. $f(-8) = (-8)^{1/3} = -2$

$\lim_{x \to -8^+} f(x) = -2$ $\lim_{x \to -8^-} f(x) = -2$ $\lim_{x \to -8} f(x) = -2$

7. $f(9) = \sqrt{9} = 3$

$\lim_{x \to 9^+} f(x) = 3$ $\lim_{x \to 9^-} f(x) = 3$ $\lim_{x \to 9} f(x) = 3$

8. $f(0) = \sqrt{4-(0)^2} = \sqrt{4} = 2$

$\lim_{x \to 0^+} f(x) = 2$ $\lim_{x \to 0^-} f(x) = 2$ $\lim_{x \to 0} f(x) = 2$

9. $f(1) = \frac{1}{1}$

$\lim_{x \to 1^+} f(x) = \frac{1}{1} = 1$ $\lim_{x \to 1^-} f(x) = \frac{1}{1} = 1$ $\lim_{x \to 1} f(x) = \frac{1}{1} = 1$

10. $f(-1) = \frac{-1-2}{(-1)^2 - 4} = \frac{-3}{-3} = 1$

$\lim_{x \to -1^+} f(x) = 1$ $\lim_{x \to -1^-} f(x) = 1$ $\lim_{x \to -1} f(x) = 1$

11. $f(x) = \frac{1^2 - 4}{1-2} = \frac{3}{1} = 3$

$\lim_{x \to 1^+} f(x) = 3$ $\lim_{x \to 1^-} f(x) = 3$ $\lim_{x \to 1} f(x) = 3$

12. $c = 0$:

$f(0) = -2(0) + 2 = 2$ because 0 is in the domain of Branch 1.

$\lim_{x \to 0^+} f(x) = 2$ $\lim_{x \to 0} f(x) = 2$

$c = 4$:

$f(4) = 4 + 3 = 7$ because ***4*** is in the domain of Branch 2.

$\lim_{x \to 4^+} f(x) = 7$ $\lim_{x \to 4^-} f(x) = 7$ $\lim_{x \to 4} f(x) = 7$

13. $f(0) = \sqrt{0} = 0$

$\lim_{x \to 0^+} f(x) = \sqrt{0} = 0$

$\lim_{x \to 0^-} f(x)$ ***does not exist (DNE)*** because there is NO (real) y-value for the x-values as we approach ***0*** from the left!

$\lim_{x \to 0} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit!

14. $c = -2$:

$f(-2) = \sqrt{4 - (-2)^2} = 0$

$\lim_{x \to -2^+} f(x) = \sqrt{4 - (-2)^2} = 0$

$\lim_{x \to -2^-} f(x)$ ***does not exist*** because there is NO (real) y-value for the x-values as we approach ***-2*** from the left!

$\lim_{x \to -2} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit!

$c = 2$:

$f(2) = \sqrt{4 - (2)^2} = 0$

$\lim_{x \to 2^+} f(x)$ ***does not exist*** because there is NO (real) y-value as we approach ***2*** from the right!

$$\lim_{x \to 2^-} f(x) = \sqrt{4-(-2)^2} = 0$$

$\lim_{x \to 2} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit!

15. $f(0) = \frac{1}{0}$ ***is undefined*** (There is a vertical asymptote at x=0 because the function is reduced to lowest terms!)

$\lim_{x \to 0^+} f(x)$ ***does not exist (DNE)*** because we DO NOT approach a specific y-value as we approach ***0*** from the right.

$\lim_{x \to 0^-} f(x)$ ***does not exist (DNE)*** because we DO NOT approach a specific y-value as we approach ***0*** from the left.

NOTE: Actually, as we approach *0* from the right and left along the x-axis, the y-values are getting infinitely large. However, infinity is NOT considered a SPECIFIC y-value.

$\lim_{x \to 0} f(x)$ ***does not exist (DNE)*** because the right-sided and left-sided limits do not exist.

16. $f(-2) = \frac{-2-2}{(-2)^2-4} = \frac{-4}{0}$ ***is undefined*** (There is a vertical asymptote at x = - 2)

$\lim_{x \to -2^+} f(x)$ ***does not exist (DNE)*** because we DO NOT approach a specific y-value as we approach ***-2*** from the right.

$\lim_{x \to -2^-} f(x)$ ***does not exist (DNE)*** because we DO NOT approach a specific y-value as we approach ***-2*** from the left.

$\lim_{x \to -2} f(x)$ ***does not exist (DNE)*** because the right-sided and left-sided limits do not exist.

17. $f(2)=\frac{2-2}{(2)^2-4}=\frac{0}{0}$ ***is undefined*** (There is a hole at x = 2)

$\lim\limits_{x\to 2^+} f(x)=\frac{1}{4}$ $\lim\limits_{x\to 2^-} f(x)=\frac{1}{4}$ $\lim\limits_{x\to 2} f(x)=\frac{1}{4}$

18. $f(2)=\frac{2^2-4}{2-2}=\frac{0}{0}$ (There is a hole at x=2!)

$\lim\limits_{x\to 2^+} f(x)=4$ $\lim\limits_{x\to 2^-} f(x)=4$ $\lim\limits_{x\to 2} f(x)=4$

19. $f(2)=8$

$\lim\limits_{x\to 2^+} f(x)=4$ $\lim\limits_{x\to 2^-} f(x)=4$ $\lim\limits_{x\to 2} f(x)=4$

Please note that The Limit and the One-Sided Limits DO NOT equal *8*! That is because the second branch of this piece-wise defined function consists only of a point and "approaching" is NOT possible. We can only "approach" using the first branch!

20. $f(2)$ ***is undefined*** because ***2*** is not in the domain of the function.

$\lim\limits_{x\to 2^+} f(x)=5$ $\lim\limits_{x\to 2^-} f(x)=-2$

$\lim\limits_{x\to 2} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit.

21. $f(2)=-2(2)+2=-2$ because ***2*** is in the domain of Branch 1.

$\lim\limits_{x\to 2^+} f(x)=5$ $\lim\limits_{x\to 2^-} f(x)=-2$

$\lim\limits_{x\to 2} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit.

22. $f(2) = -2 + 3 = 1$ because ***2*** is in the domain of Branch 2.

$$\lim_{x \to 2^+} f(x) = 5 \qquad \lim_{x \to 2^-} f(x) = -2$$

$\lim_{x \to 2} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit.

23. $f(2) = 3$ because ***2*** is in the domain of Branch 3.

$$\lim_{x \to 2^+} f(x) = 5 \qquad \lim_{x \to 2^-} f(x) = -2$$

$\lim_{x \to 2} f(x)$ ***does not exist (DNE)*** because the right-sided limit DOES NOT equal the left-sided limit.

Please note that The Limit and the One-Sided Limits DO NOT equal *3*! That is because the third branch of this piece-wise defined function consists only of a point and "approaching" is NOT possible. We can only "approach" using the first and second branches!