$\lim_{x \to \infty} \int_{2}^{3} \frac{1}{dx} \, dy$

CONCAVITY AND INFLECTION POINTS

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Problem 1:

Given $f(x) = 2x^3 + x^2 - 20x + 4$ with domain $(-\infty, \infty)$, find

- the coordinates of any *inflection points* on the graph of the function
- the intervals over which the function is concave up and concave down

Problem 2:

Given $f(x) = x^{2/3} - 1$ with domain $(-\infty, \infty)$, find

- the coordinates of any *inflection points* on the graph of the function
- the intervals over which the function is concave up and concave down

Problem 3:

Given $f(x) = \sqrt[3]{x^2 - x - 2}$ with domain $(-\infty, \infty)$, find

- the coordinates of any *inflection points* on the graph of the function
- the intervals over which the function is concave up and concave down

Problem 4:

Given $f(x) = x\sqrt{9 - x^2}$ with domain $[-3, 3]$, find

- the coordinates of any *inflection points* on the graph of the function
- the intervals over which the function is concave up and concave down

Problem 5:

Given $f(x) = x\sqrt{2} - 2\cos x$ with restricted domain $[-2\pi, 2\pi]$, find

- the x-coordinates ONLY of any *inflection points* on the graph of the function
- the intervals over which the function is concave up and concave down

Problem 6:

Given $f(x) = (x+2)^3 - 4$ with domain $(-\infty, \infty)$, find

- the coordinates of any *inflection points* on the graph of the function
- the intervals over which the function is concave up and concave down

SOLUTIONS

You can find detailed solutions below the link for this problem set!

1. Inflection Point at $[-\frac{1}{6}, f(-\frac{1}{6})] = (-\frac{1}{6}, \frac{397}{54})$ The interval over which the graph of the function is concave up is $(-\frac{1}{6}, \infty)$. The interval over which the graph of the function is concave down is $(-\infty, -\frac{1}{6})$.	2. No Inflection Points. The graph is never concave up. The intervals over which the graph of the function is concave down are $(-\infty, 0)$ and $(0, \infty)$.
3. Inflection Points at $[-1, f(-1)] = (-1, 0)$ and $[2, f(2)] = (2, 0)$ The interval over which the graph of the function is concave up is $(-1, 2)$. The intervals over which the graph of the function is concave down are $(-\infty, -1)$ and $(2, \infty)$.	4. Inflection Point at $[0, f(0)] = (0, 0)$ The interval over which the graph of the function is concave up is $[-3, 0)$. The interval over which the graph of the function is concave down is $(0, 3]$.
5. x-coordinates of the Inflection Points: $\frac{-3\pi}{2}$, $\frac{-\pi}{2}$, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$ The intervals over which the graph of the function is concave up are $\left[-2\pi, \frac{-3\pi}{2}\right)$, $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, and $\left(\frac{3\pi}{2}, 2\pi\right]$. The intervals over which the graph of the function is concave down are $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$	6. Inflection Point at $[-2, f(-2)] = (-2, -4)$. The interval over which the graph of the function is concave up is $(-2, \infty)$. The interval over which the graph of the function is concave down is $(-\infty, 2)$.