$$\lim_{x \to \infty} \int_{2}^{3} \frac{1}{dx}\frac{1}{dx}\,dy$$

INDEFINITE INTEGRALS AND ANTIDERIVATIVES OF SOME TRANSCENDENTAL FUNCTIONS

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As a review

- when you see $\displaystyle\int_{a}^{b} f(x)\,dx$ (the definite integral) you are asked to find a limiting value of the function f

- when you see $\displaystyle\int f(x)\,dx$ (the indefinite integral) you are asked to find the family of antiderivatives of the function f

In this unit, we will practice finding the *antiderivatives F* of given trigonometric functions *f* so that we can ultimately use the *Fundamental Theorem of Calculus* to find *definite integrals*.

More Basic Integration Formulas (Rules for finding Antiderivatives)

1. $$\int a^{x}\,dx = \frac{1}{\ln a}\cdot a^{x} + C$$

Special Case: $$\int e^{x}\,dx = e^{x} + C$$

2. $$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \ln|x| + C$$

Note we have to use the natural logarithm of the absolute value of *x* because this function is not defined for negative values!

3. $$\int \cos x\,dx = \sin x + C$$

4. $$\int \sec^{2} x\,dx = \tan x + C$$

5. $$\int \sec x \tan x\,dx = \sec x + C$$

6. $\int \sin x \, dx = -\cos x + C$

7. $\int \csc^2 x \, dx = -\cot x + C$

8. $\int \csc x \cot x \, dx = -\csc x + C$

Procedures for Fitting Some Integrands to the Basic Integration Formulas

a. use the Reciprocal Identities

$\sec x = \dfrac{1}{\cos x}$ or $\cos x = \dfrac{1}{\sec x}$

$\csc x = \dfrac{1}{\sin x}$ or $\sin x = \dfrac{1}{\csc x}$

$\cot x = \dfrac{1}{\tan x}$ or $\tan x = \dfrac{1}{\cot x}$

b. use the Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x}$

$\cot x = \dfrac{\cos x}{\sin x}$

c. use the Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

The basic integration formulas discussed previously are limited in scope. They do not, for example, allow us to integrate certain products or trigonometric functions containing multiple angles.

Therefore, we will now practice finding some antiderivatives using the **Chain Rule for Integration**.

The Chain Rule for Integration

$$\int f[g(x)]g'(x)\,dx = F[g(x)] + C$$

This rule is usually restated with a variable substitution as follows:

Let $u = g(x)$ be a function of **x**. This implies that $\dfrac{du}{dx} = g'(x)$ and

$$du = g'(x)\,dx.$$

then $$\int f(u)\,du = F(u) + C$$

The variable u is usually used in substitution!

WHY IS SUBSTITUTION NECESSARY?

Using substitution allows us to decide whether or not we can use the *Chain Rule for Integration*. Since the *g' (x)* factor in the integrand is not always readily apparent, the *du*-equation will immediately point out the *g' (x)* factor to you. If they match, except maybe for a difference in the constant factor, most likely the *Chain Rule for Integration* can be used.

Guidelines for Using Substitution

1. Write your integrand as a product, if necessary.

2. Decide which expression in the integrand to substitute with **u**.

3. Compute **du**.

    a. If the right side of the *du*-equation matches the remaining factor(s) in the integrand (the ones not used for **u**), except maybe for a difference in the constant

    factor, rewrite the integral as $\int f(u)\,du$ and find the family of antiderivatives using

    $\int f(u)\,du = F(u) + C$.

    b. If the right side of the *du*-equation DOES NOT match the remaining factor, try a different integration procedure.

4. If 3.a. applies, change the antiderivative back to the original variables.

NOTE: Before using substitution you should ALWAYS try to use the basic integration formulas discussed previously.

Problem 1:

Find the antiderivative for $\int \dfrac{3x^4 - 2x^3 + x^2 - 1}{3x^2}\, dx$.

Let's write the fraction as a product and factor out the constant.

$$\tfrac{1}{3}\int (3x^4 - 2x^3 + x^2 - 1)x^{-2}\, dx$$

then

$$\tfrac{1}{3}\int (3x^2 - 2x + x^{-1} - x^{-2})\, dx$$

and $\tfrac{1}{3}x^3 - \tfrac{1}{3}x^2 - \tfrac{1}{3}\ln|x| + \tfrac{1}{3}x^{-1} + C$

Therefore, the antiderivative is
$$F(x) = \frac{1}{x}x^3 - \frac{1}{3}x^2 - \frac{1}{3}\ln|x| + \frac{1}{3}x^{-1} + C$$

Problem 2:

Integrate $\int 2(5^x)\, dx$. Note that "integrate" actually means to find the antiderivative for the

function $f(x) = 2(5^x)$!!!

Let's first factor out the constant

$2\int (5^x)\, dx$ and we find the antiderivative to be
$$F(x) = \frac{2(5^x)}{\ln 5} + C$$

Problem 3:

Evaluate $\int \frac{1}{4 \sec x} dx$. Note that "evaluate" actually means to find the antiderivative for the function $f(x) = \frac{1}{4 \sec x}$!!!

Here we will use a Reciprocal Identity!

$$\int \frac{1}{4 \sec x} dx = \int \tfrac{1}{4} \cos x \, dx$$

$$= \tfrac{1}{4} \sin x + C$$

Therefore, the antiderivative is $F(x) = \frac{1}{4} \sin x + C$

Problem 4:

Evaluate $\int \frac{1}{\sin^2 x} dx$.

Again we will use a Reciprocal Identity!

$$\int \frac{1}{\sin^2 x} dx = \int \csc^2 x \, dx$$

$$= -\cot x + C$$

Therefore, the antiderivative is $F(x) = -\cot x + C$

Problem 5:

Evaluate $\int \frac{\tan z}{\cos z} dz$.

Again we will use a Reciprocal Identity!

$$\int \frac{\tan z}{\cos z} dz = \int \sec z \tan z \, dz$$

$$= \sec z + C$$

Therefore, the antiderivative is $F(z) = \sec z + C$

Problem 6:

Evaluate $\displaystyle\int \frac{\sin^2 v}{1 - \sin^2 v}\, dv$

Here we will use two different Pythagorean Identities!

$$\int \frac{\sin^2 v}{1 - \sin^2 v}\, dv = \int \frac{\sin^2 v}{\cos^2 v}\, dv$$

$$= \int \tan^2 v\, dv$$

$$= \int (\sec^2 v - 1)\, dv$$

$$= \tan v - v + C$$

Therefore, the antiderivative is $F(v) = \tan v - v + C$

Problem 7:

Evaluate $\displaystyle\int (4 + 4\tan^2 x)\, dx$

Here we will use a Pythagorean Identity!

$$\int (4 + 4\tan^2 x)\, dx = \int 4(1 + \tan^2 x)\, dx$$

$$= \int 4\sec^2 x\, dx$$

$$= 4\tan x + C$$

Therefore, the antiderivative is $F(x) = 4\tan x + C$

Problem 8:

Evaluate $\displaystyle\int \frac{\csc\theta\cos\theta}{\sin\theta}\,d\theta$

Here we will use a Quotient Identity!

$$\int \frac{\csc\theta\cos\theta}{\sin\theta}\,d\theta = \int \csc\theta\cot\theta\,d\theta$$

$$= -\csc\theta + C$$

Therefore, the antiderivative is $F(\theta) = -\csc\theta + C$

Problem 9:

Evaluate $\displaystyle\int \sec y(\tan y - \sec y)\,dy$

$$\int \sec y(\tan y - \sec y)\,dy = \int (\sec y\tan y - \sec^2 y)\,dy$$

$$= \sec y - \tan y + C$$

Therefore, the antiderivative is $F(y) = \sec y - \tan y + C$

Problem 10:

Evaluate $\displaystyle\int (\tan x - \sec x)^2\,dx$

Here we will use a Pythagorean Identity!

$$\int (\tan x - \sec x)^2\,dx = \int (\tan^2 x - 2\sec x\tan x + \sec^2 x)\,dx$$

$$= \int \left[(\sec^2 x - 1) - 2\sec x\tan x + \sec^2 x\right]dx$$

$$= \int (2\sec^2 x - 2\sec x\tan x - 1)\,dx$$

$$= 2\tan x - 2\sec x - x + C$$

Therefore, the antiderivative is $F(x) = 2\tan x - 2\sec x - x + C$

Problem 11:

Evaluate
$$\int (1 + \cot^2 x)\,dx$$

Here we will use a Pythagorean Identity!

$$\int (1 + \cot^2 x)\,dx = \int \csc^2 x\,dx$$
$$= -\cot x + C$$

Therefore, the antiderivative is $F(x) = -\cot x + C$

Problem 12:

Solve the differential equation $f'(x) = \csc x \cot x + x$ subject to the initial condition
$$f\left(\frac{\pi}{2}\right) = -1$$
.

This means that we must find the antiderivative $f(x)$ and given the initial condition we can then find the value for the constant of integration C.

$$f(x) = \int (\csc x \cot x + x)\,dx = -\csc x + \tfrac{1}{2}x^2 + C$$

Now we know that $f(x) = -\csc x + \tfrac{1}{2}x^2 + C$.

By using the initial condition $f\left(\frac{\pi}{2}\right) = -1$, we can determine the constant of integration.

That is,
$$f\left(\frac{\pi}{2}\right) = -\csc\left(\frac{\pi}{2}\right) + \tfrac{1}{2}\left(\frac{\pi}{2}\right)^2 + C = -1$$

then
$$-1 + \frac{\pi^2}{8} + C = -1$$

and we find that
$$C = -\frac{\pi^2}{8}$$

Therefore, the solution to the differential equation $f'(x) = \csc x \cot x + x$ is
$$f(x) = -\csc x + \frac{1}{2}x^2 - \frac{\pi^2}{8}$$

Problem 13:

Solve the differential equation $f''(x) = 2\cos x - 5\sin x$ subject to the initial conditions $f'(\pi) = 3$ and $f(\pi) = 2 + 6\pi$.

This means that we must find the antiderivative $f(x)$ and given the initial conditions we can then find the value for the constant of integration C. Since a second derivative is given, we first have to find $f'(x)$.

$$f'(x) = \int (2\cos x - 5\sin x)\,dx = 2\sin x + 5\cos x + C$$

Now we know that $f'(x) = 2\sin x + 5\cos x + C$.

By using the initial condition $f'(\pi) = 3$, we can determine the constant of integration.

That is, $f'(\pi) = 2\sin\pi + 5\cos\pi + C = 3$

then $2(0) + 5(-1) + C = 3$

and we find that $C = 8$

Now we know that $f'(\pi) = 2\sin\pi + 5\cos\pi + 8$.

Next we have to find $f(x)$ by finding the antiderivative of $f'(\pi) = 2\sin\pi + 5\cos\pi + 8$.

$$f(\pi) = \int (2\sin\pi + 5\cos\pi + 8)\,dx = -2\cos x + 5\sin x + 8x + C$$

Now we know that $f(x) = -2\cos x + 5\sin x + 8x + C$.

By using the initial condition $f(\pi) = 2 + 6\pi$, we can determine the constant of integration.

That is, $f(\pi) = -2\cos\pi + 5\sin\pi + 8\pi + C = 2 + 6\pi$

then $-2(-1) + 5(0) + 8\pi + C = 2 + 6\pi$

and we find that $C = -2\pi$.

Therefore, the solution to the differential equation

$$f''(x) = 2\cos x - 5\sin x$$ is

$$f(x) = -2\cos x + 5\sin x + 8x - 2\pi.$$

Problem 14:

Evaluate $\int 4\cos 4x \, dx$

Reasons for trying to use the *Chain Rule*:

1. The trigonometric expression matches one of the basic trigonometric integration formulas!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Recall:

Let **u** be a function of **x**. If $f(x) = \sin u$, then $f'(x) = u' \cos u$.

Before we can proceed with the *Chain Rule,* we must check to see if **u'** is the ONLY other factor of the integrand. This is done with the help of *u*-substitution.

$u = 4x$

$du/dx = 4$

$du = 4\, dx$

Since the right side of the *du*-equation matches the remaining factors **4** and **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$\int \cos u \, du = \sin u + C$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$\int 4\cos 4x \, dx = \sin 4x + C$

Therefore, the antiderivative is $F(x) = \sin 4x + C$

How come there is no factor 4 in the antiderivative? Where did it go?

To explain this, let's find the derivative of $F(x) = \sin 4x + C$.

Using the *Chain Rule for Differentiation*, we let **u = 4x**,

which implies that **u' = 4**.

Therefore, $F'(x) = 4 \cos 4x$!!!

Here we see that the factor 4 "appears" in the derivative due to the make-up of the Chain Rule for Differentiation. Therefore, when undoing differentiation with the Chain Rule of Integration the factor 4 must "vanish" again !!!!!!

Problem 15:

Evaluate $\int 8 \cos 4x \, dx$

This integrand is very similar to the one in Problem 14. The only difference is that it contains a constant factor **8** instead of a constant factor **4**.

However, we can still attempt to use the *Chain Rule for Integration* to evaluate the integral. Using *u*-substitution, we find

$$u = 4x$$

$$du/dx = 4$$

$$du = 4\,dx$$

The right side of the *du*-equation does NOT quite match the remaining factors of the integrand because the constant factor of the integrand is **8**. This is easily fixed by multiplying both sides of the *du*-equation by **2**.

$$2du = 8\,dx$$

NOTE: You are allowed to multiply du by a constant. HOWEVER, YOU ARE NOT ALLOWED TO MULTIPLY du BY A VARIABLE !!!

Since the right side of the *du*-equation matches the remaining factors **8** and **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$$2 \int \cos u \, du = \sin u + C$$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int 8 \cos 4x \, dx = 2 \sin 4x + C$$

Therefore, the antiderivative is $F(x) = 2 \sin 4x + C$

Problem 16:

Evaluate
$$\int \tfrac{1}{2} \cos 4x\ dx$$

This integrand is very similar to the one in Problem 14. The only difference is that it contains a constant factor instead of a factor **8**.

Therefore, we'll attempt to use the *Chain Rule for Integration* to evaluate the integral. Using *u*-substitution, we find

$u = 4 x$

$du/dx = 4$

$du = 4\ dx$

The right side of the *du*-equation does NOT quite match the remaining factors of the integrand because the constant factor of the integrand is . This is easily fixed by multiplying both sides of the *du*-equation as follows:

⅛ $du = dx$

Since the right side of the *du*-equation matches the remaining factors and **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

⅛ $\int \cos u\ du$ = ⅛ $\sin u + C$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int \tfrac{1}{2} \cos 4x\ dx = \tfrac{1}{8} \sin 4x + C$$

$$F(x) = \frac{1}{8} \sin 4x + C$$

Therefore, the antiderivative is

Problem 17:

Evaluate $\int \cos 4x \, dx$

This integrand is very similar to the one in Problem 14. The only difference is that it does not contain a constant factor.

However, we can still attempt to use the *Chain Rule for Integration* to evaluate the integral. Using *u*-substitution, we find

$u = 4x$

$du/dx = 4$

$du = 4 \, dx$

The right side of the *du*-equation contains the remaining factor **dx** of the integrand, but the integrand does not have a constant factor **4**. This is easily fixed by multiplying both sides of the *du*-equation as follows:

$du = dx$

Since the right side of the *du*-equation matches the remaining factor **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$\int \cos u \, du = \sin u + C$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$\int \cos 4x \, dx = \frac{1}{4} \sin 4x + C$

Therefore, the antiderivative is $F(x) = \frac{1}{4} \sin 4x + C$

Problem 18:

Evaluate $\int 2x \cos 4x \; dx$

This integrand SEEMS very similar to the one in Problem 14. The only difference is that it also contains a variable factor.

However, we can still attempt to use the *Chain Rule for Integration* to evaluate the integral. Using *u*-substitution, we find

u = 4 x

du/dx = 4

du = 4 dx

The right side of the *du*-equation does not match the remaining factors of the integrand. First of all the constant factor of the integrand is **2**. Secondly, the integrand has a factor **x**, but the *du*-equation does not. Taking care of the constant factor isn't a problem as we have seen in earlier examples. However, there is NOTHING that can be done to "sneak" the factor **x** into the *du*-equation !!!

NO, you cannot multiply both sides of the *du*-equation by *x* !!! That would not be a mathematically sound procedure!

Therefore, we have to give up and find a different way (beyond the scope of this course) to evaluate the integral!

Problem 19:

Evaluate $\int \dfrac{1}{\sin(1 - 3x)\tan(1 - 3x)} \; dx$

Before we start the evaluation process, let's rewrite this integrand so that it might match a basic integration formula. Using one of the *Reciprocal Identities*, we can write

$$\int \frac{1}{\sin(1 - 3x)\tan(1 - 3x)} \; dx = \int \csc(1 - 3x)\cot(1 - 3x)\,dx$$

Reasons for trying to use the *Chain Rule*:

1. The trigonometric expression matches one of the basic trigonometric integration formulas!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Let's try the *Chain Rule of Integration*! Using *u*-substitution, we find

u = 1 - 3x

du = -3 dx

The right side of the *du*-equation contains the remaining factor **dx** of the integrand, but the integrand does not have a constant factor **-3**. This is easily fixed by multiplying both sides of the *du*-equation as follows:

-⅓ du = dx

Since the right side of the *du*-equation matches the remaining factor **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

- ⅓ ∫ csc u cot u du = ⅓ csc u + C

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int \frac{1}{\sin(1-3x)\tan(1-3x)}\ dx = \int \csc(1-3x)\cot(1-3x)\,dx = \tfrac{1}{3}\csc(1-3x)+C$$

$$F(x) = \frac{1}{3}\csc(1-3x) + C$$

Therefore, the antiderivative is

Problem 20:

$$\int 7\sec 8x\tan 8x\ dx$$

Evaluate

Reasons for trying to use the *Chain Rule*:

1. The trigonometric expression matches one of the basic trigonometric integration formulas!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Let's try the *Chain Rule of Integration*! Using *u*-substitution, we find

u = 8 x

du = 8 dx

The right side of the *du*-equation does not quite match the remaining factors of the integrand because the constant factor of the integrand is **7**. This is easily fixed by multiplying both sides of the *du*-equation as follows:

⅞ du = 7 dx

Since the right side of the *du*-equation matches the remaining factors **7** and **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

⅞ ∫ sec u tan u du = ⅞ sec u + C

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int 7 \sec 8x \tan 8x \; dx = \tfrac{7}{8} \sec 8x + C$$

Therefore, the antiderivative is

$$F(x) = \frac{7}{8} \sec 8x + C$$

Problem 21:

Evaluate

$$\int \cot^2 \left(\tfrac{x}{3}\right) dx$$

Before we start the evaluation process, let's rewrite this integrand so that it might match a basic integration formula. Using one of the *Pythagorean Identities,* we can write

$$\int \cot^2 \left(\tfrac{x}{3}\right) dx = \int \left[\csc^2 \left(\tfrac{x}{3}\right) - 1 \right] dx$$

$$= \int \csc^2 \left(\tfrac{x}{3}\right) dx - \int dx$$

Reasons for trying to use the *Chain Rule*:

1. The trigonometric expression matches one of the basic trigonometric integration formulas!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Let's try the *Chain Rule of Integration*, **however, for the first integrand ONLY!** Using *u*-substitution, we find

$$u = \tfrac{1}{3}\,x$$

$$du = \tfrac{1}{3}\,dx$$

The right side of the *du*-equation contains the remaining factor **dx** of the integrand, but the integrand does not have a constant factor of ⅓. This is easily fixed by multiplying both sides of the *du*-equation as follows:

$$3du = dx$$

Since the right side of the *du*-equation matches the remaining factor **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$$3\int csc^2 u\; du = \text{-}3\, cot\, u + C$$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int cot^2\left(\tfrac{x}{3}\right) dx = \int \left[csc^2\left(\tfrac{x}{3}\right) - 1\right] dx$$

$$= \int csc^2\left(\tfrac{x}{3}\right) dx - \int dx$$

$$= -3\, cot\left(\tfrac{x}{3}\right) - x + C$$

Therefore, the antiderivative is $F(x) = -3\, cot\left(\tfrac{x}{3}\right) - x + C$

Problem 22:

Evaluate $\displaystyle\int \frac{sec\, 2x}{cos\, 2x}\, dx$

Before we start the evaluation process, let's rewrite this integrand so that it might match a basic integration formula. Using one of the *Pythagorean Identities*, we can write

$$\int \frac{sec\, 2x}{cos\, 2x}\, dx = \int sec^2(2x)\, dx$$

Reasons for trying to use the *Chain Rule*:

1. The trigonometric expression matches one of the basic trigonometric integration formulas!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Try the *Chain Rule of Integration*. Using *u*-substitution, we find

$$u = 2x$$

$$du = 2\,dx$$

The right side of the *du*-equation contains the remaining factor **dx** of the integrand, but the integrand does not have a constant factor of **2**. This is easily fixed by multiplying both sides of the *du*-equation as follows:

$$du = dx$$

Since the right side of the *du*-equation matches the remaining factor **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$$\int sec^2 u\,du = tan\,u + C$$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int \frac{sec\,2x}{cos\,2x}\,dx = \int sec^2(2x)\,dx = \tfrac{1}{2}\,tan\,2x + C$$

Therefore, the antiderivative is
$$F(x) = \frac{1}{2}\,tan\,2x + C$$

Problem 23:

Evaluate
$$\int 3e^{3x+1}\,dx$$

Reasons for trying to use the *Chain Rule*:

1. The indefinite integral almost matches the basic exponential integration formula!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Recall:

Let **u** be a function of **x**. If $f(x) = e^u$, then $f'(x) = u'e^u$.

Before we can proceed with the *Chain Rule for Integration,* we must check to see if u' is the ONLY other factor of the integrand. This is done with the help of *u*-substitution.

$$u = 3x + 1$$

$$du = 3\ dx$$

Since the right side of the *du*-equation matches the remaining factors **3** and **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$$\int e^u\ du = e^u + C$$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int 3e^{3x+1}\ dx = e^{3x+1} + C$$

Therefore, the antiderivative is $F(x) = e^{3x+1} + C$

Problem 24:

Evaluate $\int 5xe^{-x^2}\ dx$

Reasons for trying to use the *Chain Rule*:

1. The indefinite integral almost matches the basic exponential integration formula!

2. Because the angle is "more" than just a single variable with coefficient 1, the integrand might be a derivative found through the use of the *Chain Rule for Differentiation.*

Let's try the *Chain Rule of Integration*! Using *u*-substitution, we find

$$u = -x^2$$

$$du = -2x\ dx$$

The right side of the *du*-equation does NOT quite match the remaining factors of the integrand because the constant factor of the integrand is **5**. This is easily fixed by multiplying both sides of the *du*-equation as follows:

$$-5/2\ du = 5\ dx$$

Since the right side of the *du*-equation matches the remaining factors **5** and **dx** of the integrand exactly, we are now assured that we can use the *Chain Rule* to find the family of antiderivatives.

We proceed to write the integral in terms of **u** and evaluate as follows:

$$-5/2 \int e^u \, du = -5/2 \, e^u + C$$

and since the variable of integration is **x**, we'll convert back to the original variable to find that

$$\int 5xe^{-x^2} \, dx = -\tfrac{5}{2} e^{-x^2} + C$$

Therefore, the antiderivative is
$$F(x) = -\frac{5}{2} e^{-x^2} + C$$