$\lim_{x \to \infty} \int_{2}^{3} \frac{1}{dx} dy$

SOME BASIC DIFFERENTIATION RULES

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You can find the proofs of these rules in the online textbook as a separate document.

Constant Rule:

If $f(x) = b$, then $f'(x) = 0$. Note that b is any real number!

Examples:

1. If $f(x) = 5$, then $f'(x) = 0$
2. If $f(x) = -2$, then $f'(x) = 0$
3. If $f(x) = 0$, then $f'(x) = 0$

Simple Power Rule:

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$, where n is any real number.

Examples:

1. If $f(x) = x^5$, then $f'(x) = 5x^{5-1} = 5x^4$
2. If $f(x) = x$, then $f'(x) = x^{1-1} = x^0 = 1$
3. If $f(x) = x^{1/3}$, then $f'(x) = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3}$

   and $f'(x) = \frac{1}{3x^{2/3}}$

4. If $f(x) = \sqrt{x}$ and $\sqrt{x} = x^{1/2}$,

   then $f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$

   and $f'(x) = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

5. If $f(x) = x^{-4}$,

then $f'(x) = -4x^{-4-1} = -4x^{-5}$

and $f'(x) = \frac{-4}{x^5}$

Constant Multiple Rule:

Let ***k*** be a constant and ***u*** a function of ***x***.

If $f(x) = ku$, then $f'(x) = k \cdot u'$.

Examples:

1. If $f(x) = 2x^5$, where $k = 2$, $u = x^5$,

and $u' = 5x^4$

then $f'(x) = 2(5x^4) = 10x^4$

2. If $f(x) = -\frac{5}{7}x^{2/3}$, where $k = -\frac{5}{7}$, $u = x^{2/3}$,

and $u' = \frac{2}{3}x^{-1/3}$

then $f'(x) = -\frac{5}{7}(\frac{2}{3}x^{-1/3}) = -\frac{10}{21}x^{-1/3}$

and $f'(x) = \frac{-10}{21x^{1/3}}$

Sum/Difference Rule:

Let ***u*** and ***v*** be a function of ***x***.

If $f(x) = u \pm v$, then $f'(x) = u' \pm v'$.

Examples:

1. If $f(x) = 2x^5 - 4x^6$, where $u = 2x^5$

and $v = -4x^6$, then $f'(x) = 10x^4 - 24x^5$

2. If $f(x) = x^{3/2} - 3x^{5/4} + 2x - 1$,

then by extending the *Sum/Difference Rule* we get

$$f'(x) = \frac{3}{2}x^{1/2} - \frac{15}{4}x^{1/4} + 2$$

Higher Order Derivatives

First Derivative:

$f'(x)$ or $\frac{dy}{dx}$

Second Derivative:

$f''(x)$ pronounced ***f*** *double prime of* ***x***

or $\frac{d^2y}{dx^2}$ pronounced the *second derivative of* ***y*** *with respect to* ***x***

Third Derivative:

$f'''(x)$ pronounced ***f*** *triple prime of* ***x***

or $\frac{d^3y}{dx^3}$ pronounced the *third derivative of* ***y*** *with respect to* ***x***

Fourth Derivative:

$f^{(4)}(x)$ pronounced the *fourth derivative of* ***f*** *with respect to* ***x***

or $\frac{d^4y}{dx^4}$ pronounced the *fourth derivative of* ***y*** *with respect to* ***x***

99th Derivative:

$f^{(99)}(x)$ pronounced the *99th derivative of* ***f*** *with respect to* ***x***

or $\frac{d^{99}y}{dx^{99}}$ pronounced the *99th derivative of* ***y*** *with respect to* ***x*** nth Derivative:

$f^{(n)}(x)$ pronounced the *nth derivative of* ***f*** *with respect to* ***x***

or $\frac{d^n y}{dx^n}$ pronounced the *nth derivative of* ***y*** *with respect to* ***x***

Problem 1:

Find the derivative for the following functions.

a. $f(x) = -4x^{5/4}$

$f'(x) = -4\left(\frac{5}{4}x^{5/4-1}\right)$

and $f'(x) = -5x^{1/4}$

NOTE: Whenever possible you should ALWAYS combine like terms in your solutions.

b. $f(x) = 6 - x + 2x^3 - 4x^6$

$f'(x) = 0 - x^{1-1} + 2(3x^{3-1}) - 4(6x^{6-1})$

and $f'(x) = -1 + 6x^2 - 24x^5$

c. $f(x) = -2x^3 - \sqrt[3]{x^5}$

$f(x) = -2x^3 - \sqrt[3]{x^5} = -2x^3 - x^{5/3}$

$f'(x) = -2(3x^{3-1}) - \frac{5}{3}x^{5/3-1}$

and $f'(x) = -6x^2 - \frac{5}{3}x^{2/3}$

d. $f(x) = x^{4/3}(x^2 + 3x^{2/3} - 6)$

$f(x) = x^{4/3}(x^2 + 3x^{2/3} - 6) = x^{10/3} + 3x^2 - 6x^{4/3}$

$f'(x) = \frac{10}{3}x^{7/3} + 6x - 8x^{1/3}$

e. $f(x) = (2x - 3)^2$

$f(x) = (2x - 3)^2 = 4x^2 - 12x + 9$

$f'(x) = 8x - 12$

f. $f(x) = (x^3 - 2)(2x + 1)$

$f(x) = (x^3 - 2)(2x + 1) = 2x^4 + x^3 - 4x - 2$

$f'(x) = 8x^3 + 3x^2 - 4$

Problem 2:

Differentiate the following functions. Write any negative exponents in your answer as fractions and then write as a SINGLE fraction, if necessary.

a. $f(x) = x^3 + \frac{1}{x}$

$f(x) = x^3 + \frac{1}{x} = x^3 + x^{-1}$

$f'(x) = 3x^2 + (-x^{-2}) = 3x^2 - \frac{1}{x^2}$

and $f'(x) = \dfrac{3x^4 - 1}{x^2}$

b. $f(x) = (4x)^{-2}$

$f(x) = (4x)^{-2} = 4^{-2}x^{-2} = \frac{1}{16}x^{-2}$

$f'(x) = -\frac{1}{8}x^{-3}$

and $f'(x) = \dfrac{-1}{8x^3}$

c. $$f(x) = \frac{3x^2 - 7x + 2}{x}$$

$$f(x) = \frac{3x^2 - 7x + 2}{x} = 3x - 7 + 2x^{-1}$$

$$f'(x) = 3 - 2x^{-2} = 3 - \frac{2}{x^2}$$

and $$f'(x) = \frac{3x^2 - 2}{x^2}$$

d. $$f(x) = \frac{4x^3 + 5x - 9}{2}$$

$$f(x) = \frac{4x^3 + 5x - 9}{2} = 2x^3 + \tfrac{5}{2}x - \tfrac{9}{2}$$

$$f'(x) = 6x^2 + \tfrac{5}{2}$$

and $$f'(x) = \frac{12x^2 + 5}{2}$$

e. $$f(x) = \frac{3\sqrt{x}}{x}$$

$$f(x) = \frac{3\sqrt{x}}{x} = 3x^{-1/2}$$

$$f'(x) = -\tfrac{3}{2}x^{-3/2}$$

and $$f'(x) = \frac{-3}{2x^{3/2}}$$

f. $y = \frac{4x^2 - 5}{x^3}$

$$y = \frac{4x^2 - 5}{x^3} = 4x^{-1} - 5x^{-3}$$

$$\frac{dy}{dx} = -4x^{-2} + 15x^{-4} = \frac{-4}{x^2} + \frac{15}{x^4}$$

and $\frac{dy}{dx} = \frac{-4x^2 + 15}{x^4}$

Problem 3:

Find the slope-intercept equation of the line tangent to the graph of $f(x) = 2x - 3x^{1/2}$ at the point $(9,9)$.

Use the point-slope form $y - y_1 = m(x - x_1)$ with $m = f'(9)$.

Since $f'(x) = 2 - \frac{3}{2}x^{-1/2} = 2 - \frac{3}{2\sqrt{x}}$ and $f'(9) = 2 - \frac{3}{2\sqrt{9}} = \frac{3}{2}$,

then $y - 9 = \frac{3}{2}(x - 9) = \frac{3}{2}x - \frac{27}{2} + 9 = \frac{3}{2}x - \frac{9}{2}$

and $y = \frac{3}{2}x - \frac{9}{2}$

Thus, slope-intercept equation of the line tangent to the graph of $f(x) = 2x - 3x^{1/2}$ at the point $(9,9)$ is $y = \frac{3}{2}x - \frac{9}{2}$.

Problem 4:

Find the first, second derivative, and third derivatives of the following functions:

a. Given $f(x) = -2x^{6/5}$

$$f'(x) = -\frac{12}{5}x^{1/5}$$

$$f''(x) = -\frac{12}{25}x^{-4/5}$$

$$f'''(x) = \frac{48}{125}x^{-9/5}$$

b. Given $y = 3x - 1$

$$\frac{dy}{dx} = 3$$

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{d^3y}{dx^3} = 0$$