DETAILED SOLUTIONS AND CONCEPTS - INTRODUCTION TO EXPONENTS AND ROOTS

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YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Exponents

Exponents or powers indicate how many times a number is multiplied by itself.

Following are some examples:

- 3^0 indicates ***1***. The value of any number with an exponent of *0* equals *1*. The only exception is 0. When raising it to the *0* power, its value is *undefined*.

- 3^1 indicates ***3***. It is customary in mathematics NOT to write the exponent 1.

- 3^2 indicates ***3(3) = 9***.

  NOTE: Numbers that result form other numbers being raised to a power are also referred to as **perfect powers**!

  3^2 is called an **exponential expression** and is read as "3 squared" or "three to the second power" or "three raised to the second power."

  3 is called the **base** and ***2*** is called the **exponent** or **power**. The power indicates how many times the base is supposed to be multiplied by itself.

- 3^3 indicates ***3(3)(3) = 27***. It is read as "three cubed" or "three to the third power" or "three raised to the third power."

- 3^4 indicates ***3(3)(3)(3) = 81***. It is read as "three to the fourth power" or "three raised to the fourth power."

Roots

Finding a root reverses the operation of finding a power. The **radical sign** $\sqrt{\ }$ indicates this process.

Following are some examples!

- To undo $3^4 = 81$, we write $\sqrt[4]{81} = 3$, where ***4*** is called the **index** and ***81*** is called the **radicand**. It is read as the "fourth root of 81." We call $\sqrt[4]{81}$ a **radical expression**.
- To undo $3^3 = 27$, we write $\sqrt[3]{27} = 3$. It is read as the "third root of 27" or the "cube root of 27."
- To undo $3^2 = 9$, we write $\sqrt{9} = 3$. It is read strictly as the "square root of 9." **Please note when the index is *2* it is customarily left off.**

What about $\sqrt{10}$. Here we don't immediately know a number that when multiplied by itself results in a product of 10. Usually, these radicals are evaluated using a calculator. If a calculator is not handy, we can make a rough estimate and say that the value of $\sqrt{10}$ must be slightly greater than 3 because we know that $\sqrt{9} = 3$.

Problem 1:

Evaluate 10^3.

$10(10)(10) = 1{,}000$

Problem 2:

Evaluate 2^5.

$2(2)(2)(2)(2) = 32$

Problem 3:

Evaluate 3.4^2.

$3.4(3.4) = 11.56$

Problem 4:

Evaluate $5{,}982^0$.

The value of any number with an exponent of *0* equals *1*.

$5{,}982^0 = 1$

Problem 5:

Evaluate $\mathbf{1}^{23}$.

Since 1(1)(1)(1)(1) .... (1) always equals 1 no matter how many times we multiply it by itself, we can say that

$$1^{23} = 1$$

Problem 6:

Evaluate $\mathbf{0}^{41}$.

Since 0(0)(0)(0)(0) .... () always equals 0 no matter how many times we multiply it by itself, we can say that

$$0^{41} = 0$$

Problem 7:

Evaluate $\left(\frac{1}{8}\right)^2$.

$$\frac{1}{8}\left(\frac{1}{8}\right) = \frac{1(1)}{8(8)} = \frac{1}{64}$$

Problem 8:

Evaluate $\left(\frac{2}{5}\right)^3$.

$$\frac{2}{5}\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2(2)(2)}{5(5)(5)} = \frac{8}{125}$$

Problem 9:

Evaluate $\left(\frac{1}{3}\right)^0$.

The value of any number with an exponent of 0 equals 1.

$$\left(\frac{1}{3}\right)^0 = 1$$

Problem 10:

Evaluate $\sqrt{1}$.

Since ***1(1) = 1***, no matter what the index, a radical expression containing a radicand of ***1*** always has a value of ***1***.

That is, $\sqrt{1} = 1$.

Problem 11:

Evaluate $\sqrt{0}$.

Since ***0(0) = 0***, no matter what the index, a radical expression containing a radicand of ***0*** always has a value of ***0***.

That is, $\sqrt{0} = 0$.

Problem 12:

Evaluate $\sqrt{36}$.

Since ***6(6) = 36,*** we can say that $\sqrt{36} = 6$.

Problem 13:

Evaluate $\sqrt{100}$.

Since ***10(10) = 100,*** we can say that $\sqrt{100} = 10$.

Problem 14:

Evaluate $\sqrt{81}$.

Since ***9(9) = 81*** we can say that $\sqrt{81} = 9$.

Problem 15:

Evaluate $\sqrt{400}$.

Since ***20(20) = 400,*** we can say that $\sqrt{400} = 20$.

Problem 16:

Evaluate $\sqrt{0.64}$.

We know that ***8(8) = 64***. Then ***0.8(0.8) = 0.64***. Therefore, $\sqrt{0.64} = 0.8$

Problem 17:

Considering perfect squares, find two successive decimal numbers between which the value of $\sqrt{0.69}$ is located.

NOTE: Numbers that result from other numbers being raised to the second power are also referred to as "perfect squares"!

We know that $\sqrt{0.64} = 0.8$ and $\sqrt{0.81} = 0.9$. Please note that we would consider 0.64 and 0.81 *perfect squares* because they result from two numbers, namely 0.8 and 0.9, being raised to the second power.

Therefore, the value of $\sqrt{0.69}$ must be between ***0.8*** and ***0.9***.

Problem 18:

Considering perfect squares, find two successive decimal numbers between which the value of $\sqrt{0.55}$ is located.

We know that $\sqrt{0.49} = 0.7$ and $\sqrt{0.64} = 0.8$.

Therefore, the value of $\sqrt{0.55}$ must be between ***0.7*** and ***0.8***.