DETAILED SOLUTIONS AND CONCEPTS - RATIOS AND PROPOSITIONS
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YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Ratio

A ratio is the comparison of two like quantities separated by a colon or a fraction bar.

For example,

$2:3$ or $\frac{2}{3}$. When we are discussing ratios, we say "2 to 3" and not two-thirds!

Proportion

A mathematical statement expressing the equality of two ratios.

For example,

$2:3 = 4:6$ or $\frac{2}{3} = \frac{4}{6}$.

Solving for the Variable or Unknown in Proportions

Some proportions contain an unknown number, sometimes called a variable. Most likely you will be asked to find the value of this variable. For this we will utilize the Principle of Cross-Multiplication which says the following:

If $\frac{a}{b} = \frac{c}{d}$ then $a(d) = b(c)$. This can also be written as $b(c) = a(d)$.

Problem 1:

Find the unknown in the proportion $\frac{F}{3} = \frac{9}{5}$.

We'll use cross-multiplication.

and then we multiply the digits on either end of the two cross bars. That is,

5F = 3(9)

5F = 27

Please note the following:

If the factors of a product consist of a number and a variable,

- the number is always written first.
- the multiplication between the number and the variable is implied.

Since we must find the value of ***F = 1F***, we must divide BOTH numbers on either side of the equal sign by 5.

$$\frac{5F}{5} = \frac{27}{5}$$

and $F = \frac{27}{5} = 5\frac{2}{5}$

Direct Variation

Two quantities are said to be directly proportional if an increase in one produces an increase in the other or if a decrease in one produces a decrease in the other.

Setting up a Direct Variation

- Establish two pairs of related data.
- Write one pair of data in the numerators of two ratios.
- Write the other pair of data in the denominator of two ratios.
- Form a proportion using the two ratios.

NOTE: When setting up a direct proportion in fractional form, the numerator and denominator of each ratio must contain the same type of information.

Problem 2:

A chain saw requires a mixture of 2-cycle engine oil and gasoline. According to the directions on a bottle of Oregon 2-cycle Engine Oil, 2.5 fluid ounces of oil are required for 1 gallon of gasoline. For 2.75 gallons, how many fluid ounces of oil are required?

$$\frac{2.5}{x} = \frac{1}{2.75}$$ **Note that the first ratio contains fluid ounces and the second ratio contains gallons!**

$$x = 2.5(2.75)$$

Note that a factor 1 in a product is implied and always omitted. Furthermore, it is standard practice to place the variable to the left of the equal sign.

$$x = 6.875$$

For 2.75 gallons, 6.875 fluid ounces of oil are required.

Problem 3:

In 1998, the average exchange rate between U.S. dollars and United Kingdom pounds was 1 pound to $1.6762. How many U.S. dollars equal 400 U.K. pounds?

$$\frac{1}{400} = \frac{1.6762}{x}$$ **Note that the first ratio contains U.K. pounds and the second ratio contains U.S. dollars!**

$$x = 400(1.6762)$$

$$x = 670.48$$

$670.48 equal 400 U.K. pounds.

Problem 4:

If 6 gallons of premium unleaded gasoline cost $3.72, how much would it cost to completely fill a 15-gallon tank?

$$\frac{6}{15} = \frac{3.72}{x}$$ **Note that the first ratio contains gallons and the second ratio contains dollars!**

$$6x = 15(3.72)$$

$$6x = 55.8$$

$$\frac{6x}{6} = \frac{55.8}{6}$$

$x = 9.3$

It would cost $9.30 to completely fill a 15-gallon tank.

Problem 5:

The distance between Kansas City, Missouri, and Denver, Colorado is 600 miles. On a certain wall map, this is represented by a length of 2.4 feet. On this map, how many feet would there be between Memphis and Philadelphia, two cities that are actually 1,000 miles apart?

$$\frac{600}{1000} = \frac{2.4}{x}$$ **Note that the first ratio contains actual miles and the second ratio contains "map miles"!**

$600x = 1000(2.4)$

$600x = 2400$

$$\frac{600x}{600} = \frac{2400}{600}$$

$x = 4$

On this wall map there are 4 feet between Memphis and Philadelphia.

Problem 6:

The safe fluoride to water ratio is between 0.7 and 1.2 parts per million gallons of water. What is the range of fluoride that could be added to 200,000 gallons of water?

Lower bound:

$$\frac{0.7}{x} = \frac{1,000,000}{200,000}$$

$1,000,000x = 0.7(200,000)$

$1,000,000x = 140,000$

$$\frac{1000000x}{1000000} = \frac{140000}{1000000}$$

$x = 0.14$

Upper bound:

$$\frac{1.2}{x} = \frac{1{,}000{,}000}{200{,}000}$$

$$1{,}000{,}000x = 1.2(200{,}000)$$

$$1{,}000{,}000x = 240{,}000$$

$$\frac{1000000x}{1000000} = \frac{240000}{1000000}$$

$$x = 0.24$$

The range of fluoride that could be added to 200,000 gallons of water is between 0.14 and 0.24 parts.

Problem 7:

A patient is to receive 125 mg of Ampicillin. A nurse has a solution for injection that contains 1 g of Ampicillin per 4 mL of solution. How many mL of solution should be injected?

$$\frac{1}{0.125} = \frac{4}{x}$$

$$x = 0.125(4)$$

and $x = 0.5$

0.5 mL of solution should be injected.

Problem 8:

A disinfecting solution is mixed at a ratio of 2 parts of alcohol to 5 parts of distilled water. If the solution has 0.5 liters of water, how many milliliters of alcohol does it contain?

$$\frac{2}{x} = \frac{5}{0.5}$$

$$5x = 2(0.5)$$

$$5x = 1$$

$$\frac{5x}{5} = \frac{1}{5}$$

and $x = 0.2$

The solutions contains 0.2 L or 200 mL of alcohol.

Problem 9:

A factory worker can assemble 24 watches in 45 minutes. If her pace stays constant, how many watches can she assemble every 15 minutes?

Let's use the letter W to represent the number of watches.

$$\frac{D}{25} = \frac{1.5}{2}$$

After cross-multiplication we get

$45W = 24(15)$

$45W = 360$

Next, we will "isolate" the letter W by dividing both sides of the equation by 45.

We get $W = 8$.

That is, the factory worker can assemble 8 watches in 15 minutes.

Problem 10:

On a certain map, 2 inches represents 25 miles. If the distance between Humarock and Cody measures 1.5 inches on the map, how many miles are the cities apart?

Let's use the letter D to represent the distance between Humarock and Cody.

$$\frac{D}{25} = \frac{1.5}{2}$$

After cross-multiplication we get

$2D = 25(1.5)$

$2D = 37.5$

Next, we will "isolate" the letter D by dividing both sides of the equation by 2.

We get ***D = 18.75***.

That is, the actual distance between Humarock and Coy is 18.75 miles.