DETAILED SOLUTIONS AND CONCEPTS - OPERATIONS ON SYSTEMS OF MEASUREMENT

Prepared by Ingrid Stewart, Ph.D., College of Southern Nevada
Please Send Questions and Comments to ingrid.stewart@csn.edu.
Thank you!

YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

ADDITION AND SUBTRACTION

Rule for U.S. Customary Measures:

- Align the numbers in columns by units of measure.
- Add other digits in each column starting with the smallest unit of measure.
- Express the sum in standard notation.

Rule for Metric Measures:

- If there is more than one unit of measure, change all units to the largest common unit of measure.
- Add the numbers associated with the units of measure.
- Give the answer the largest common unit of measure.

Problem 1:

Find the sum of ***4 ft + 5 in***.

$$\begin{array}{r} 4 \text{ ft } 0 \text{ in} \\ +\ 0 \text{ ft } 5 \text{ in} \\ \hline 4 \text{ ft } 5 \text{ in} \end{array}$$

We are required to write the sum in standard notation.

We observe that ***4 ft = 3 ft + 1 ft = 1 yd 1ft***

Therefore, ***4 ft 5 in = 1 yd 1 ft 5 in***

Problem 2:

Find the sum of ***7 lb 8 oz + 3 lb 12 oz***. Write your answer in standard notation.

$$\begin{array}{r} 7\text{ lb }\ 8\text{ oz} \\ +\ 3\text{ lb }12\text{ oz} \\ \hline 10\text{ lb }20\text{ oz} \end{array}$$

We are required to write the sum in standard notation.

We observe that ***20 oz = 16 oz + 4 oz = 1 lb 4 oz***

Therefore, ***10 lb 20 oz = 10 lb + 1 lb 4 oz***

= 11 lb 4 oz

Problem 3:

Find the sum of ***681 mL + 7.8 L***. Write your answer in standard notation.

Here we will change ***681 mL*** to ***L*** by using the unit ratio $\frac{1\,L}{1000\,mL}$.

$$\frac{681\,mL}{1}\left(\frac{1\,L}{1000\,mL}\right)=\frac{681(1)\,L}{1(1000)}$$
$$=0.681\,L$$

Now we can add!

$$\begin{array}{r} 7.800\text{ L} \\ +\ 0.681\text{ L} \\ \hline 8.4\,81\text{ L} \end{array}$$

The sum equals ***8.481 L***.

Problem 4:

Find the sum of ***7479 mm + 285 cm + 27.6 m***

Here we must change all units to the largest common unit of measure which is meters.
We will use the following unit ratios:

$$\frac{1\,m}{1000\,mm} \text{ and } \frac{1\,m}{100\,cm}$$

$$\frac{7479\,\cancel{mm}}{\cancel{1}}\left(\frac{\cancel{1}\,m}{1000\,\cancel{mm}}\right) = 7.479\,m$$

$$\frac{285\,\cancel{cm}}{\cancel{1}}\left(\frac{\cancel{1}\,m}{100\,\cancel{cm}}\right) = 2.85\,m$$

Now we can add!

$$\begin{array}{r} 7.479\ m \\ 2.850\ m \\ +\ 27.600\ m \\ \hline 37.929\ m \end{array}$$

SUBTRACTION

Rule for U.S. Customary Measures:

- Align the numbers in columns by units of measure.
- Subtract the digits in each column starting with the smallest unit of measure.
- Regroup whenever subtracting a larger digit from a smaller one.
- Express the difference in standard notation.

Rule for Metric Measures:

- If there is more than one unit of measure, change all units to the largest common unit of measure.
- Subtract the numbers associated with the units of measure.
- Give the answer the largest common unit of measure.

Problem 5:

Find the difference of ***4 ft 8 in − 3 ft 2 in***.

$$\begin{array}{r} 4\ ft\ 8\ in \\ -\ \underline{3\ ft\ 2\ in} \\ 1\ ft\ 6\ in \end{array}$$

The difference is already in standard notation and equals ***1 ft 6 in***.

Problem 6:

Find the difference of ***6 lb 7 oz − 2 lb 9 oz***.

$$\begin{array}{r} 6\text{ lb }7\text{ oz} \\ -\,2\text{ lb }9\text{ oz} \\ \hline \end{array}$$

In the *ounce* column we notice that we are subtracting a larger number from a smaller one. We must regroup by borrowing one pound from the *pound* column and place its *ounce* equivalent (1 lb = 16 oz) in the *ounce* column.

$$\begin{array}{r} 5\text{ lb }23\text{ oz} \\ -\,2\text{ lb }\ \ 9\text{ oz} \\ 3\text{ lb }14\text{ oz} \end{array}$$

The difference is already in standard notation and equals ***3 lb 14 oz***.

Problem 7:

Find the difference of ***3.786 kg − 455 mg***.

Here we must change to the largest common unit of measure which is kilograms. We will use the following unit ratios:

$$\frac{1\text{ g}}{1000\text{ mg}} \text{ and } \frac{1\text{ kg}}{1000\text{ g}}$$

$$\frac{455\text{ mg}}{1}\left(\frac{1\text{ g}}{1000\text{ mg}}\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right) = \frac{455(1)(1)\text{ kg}}{1000(1000)} = 0.000455\text{ kg}$$

Now we can subtract!

$$\begin{array}{r} 3.786000\text{ kg} \\ -\,0.000455\text{ kg} \\ \hline \end{array}$$

$$3.785545\text{ kg}$$

The difference equals ***3.785545 kg***.

MULTIPLICATION BY A NUMBER

Rule for U.S. Customary and Metric Measures:

- Multiply the number associated with each unit of measure by the multiplier.
- Express the difference in standard notation.

Problem 8:

Multiply ***13 gal 1 qt*** by ***6***. Write your answer in standard notation.

13 gal 1 qt
× 6
_____
78 gal 6 qt

We are required to write the answer in standard notation.

We observe that ***6 qt = 4 qt + 2 qt***

= 1 gal + 2 qt

The final product is ***78 gal + 1 gal 2 qt = 79 gal 2 qt***

Problem 9:

Multiply ***3.8 m*** by ***9***. Write your answer in standard notation.

3.8 m
×9
_____
34.2 m

The product equals ***34.2 m***.

MULTIPLICATION BY A UNIT OF MEASURE

Rule:

- Convert both units of measure to the same unit of measure, if they are different.
- Multiply the numbers associated with each unit of measure.
- Multiply the units of measure.

Problem 10:

Multiply ***3 mi*** by ***5 mi***

3(5)(mi)(mi) = 15 mi^2

Please note that when we multiply two like units of measure, we get a **square measure**.

The product equals ***15 mi^2***.

Problem 11:

Multiply ***4 ft*** by ***10 in***

First we must convert 4 ft to inches.

$$\frac{4\,ft}{1}\left(\frac{12\,in}{1\,ft}\right)=\frac{4(12)\,in}{1}$$
$$=48\,in$$

Now we can multiply.

$$48(10)(in)(in)=480\,in^2$$

The product equals $480\,in^2$.

Problem 12:

Multiply ***4 m*** by ***1 m*** by ***5 m***

$$4(1)(5)(m)(m)(m)=20\,m^3$$

Please note that when we multiply three like units of measure, we get a **cubic measure**.

The product equals $20\,m^3$.

DIVISION BY A NUMBER

Rule:

- Divide the number associated with each unit of measure by the divisor.
- Express the difference in standard notation.

Problem 13:

Divide ***12 gal 2 qt*** by ***6***.

$$6\overline{)\,12\,gal\;\;2\,qt}\;\;\;2\,gal\;\tfrac{1}{3}\,qt$$

The quotient is already in standard notation and equals $2\,gal\;\frac{1}{3}\,qt$.

Problem 14:

Divide ***11 gal 3 qt 1 pt*** by ***3***.

$$3\overline{)\begin{array}{cccc} \frac{11}{3}=3\frac{2}{3}\,gal & 1\,qt & \frac{1}{3}\,pt \\ 11\,gal & 3\,qt & 1\,pt \end{array}}$$

The quotient is $3\frac{2}{3}\,gal\ 1\,qt\ \frac{1}{3}\,pt$.

We are required to write the answer in standard notation. However, in standard notation the numbers associated with the units of measure are whole numbers unless it is the number associated with the smallest unit of measure.

Starting with the largest unit of measure (gallons), we will change the fractional part of the number associated with gallons to the next smaller unit of measure, which is quarts. That is,

$$\frac{2\,gal}{3}\left(\frac{4\,qt}{1\,gal}\right)=\frac{2(4)\,qt}{3}$$

$$=\frac{8\,qt}{3}$$

$$=2\frac{2}{3}\,qt$$

Now we can write the quotient as $3\,gal\ 3\frac{2}{3}\,qt\ \frac{1}{3}\,pt$.

Next, we'll change the fractional part of the number associated with quarts to the next smaller unit of measure, which is pints. That is,

$$\frac{2\,qt}{3}\left(\frac{2\,pt}{1\,qt}\right)=\frac{2(2)\,pt}{3}$$

$$=\frac{4\,pt}{3}$$

$$=\frac{4}{3}\,pt$$

$$=1\frac{1}{3}\,pt$$

Now we can write the quotient as $3\,gal\,3\,qt\,1\frac{2}{3}\,pt$.

But we are not done yet. Pint is not yet the smallest unit of measure. Therefore, we'll change the fractional part of the number associated with pints to the smallest unit of measure, which is ounces. That is,

$$\frac{2\,pt}{3}\left(\frac{16\,oz}{1\,pt}\right)=\frac{2(16)oz}{3}$$
$$=\frac{32}{3}oz$$
$$=10\frac{2}{3}oz$$

Finally, we can write the quotient as $3\,gal\;3\,qt\;1\,pt\;10\frac{2}{3}oz$. Please note that the fraction is associated with the smallest unit of measure, which is acceptable according to the Standard Notation Rule.

Problem 15:

Divide ***150 cm*** by ***6***.

$$6\overline{)150\,cm}\;\;25\,cm$$

The quotient equals ***25 cm***.

DIVISION BY A UNIT OF MEASURE

Rule:

- Convert both units of measure to the same unit, if they are different.
- Divide the numbers associated with each unit of measure. Write this division as a fraction!
- Divide the units of measure.

Problem 16:

Divide ***12 gal*** by ***6 gal***.

$$\frac{\overset{2}{\cancel{12}}\,gal}{\underset{1}{\cancel{6}}\,gal}=2$$

The quotient equals ***2***.

Problem 17:

Divide ***3.6 lb*** by ***9 lb***. Write your answer in standard notation.

$$\frac{\overset{0.4}{\cancel{3.6\ lb}}}{\underset{1}{\cancel{9\ lb}}} = 0.4$$

The quotient equals ***0.4***.

Problem 18:

Divide ***340 kg*** by ***5 kg***.

$$\frac{\overset{68}{\cancel{340\ kg}}}{\underset{1}{\cancel{5\ kg}}} = 68$$

The quotient equals ***68***.

Some approximate U. S. Customary and Metric Comparisons

NOTE: The symbol ≈ means "approximately equal to".

1 cm ≈ 0.3937 in

1 m ≈ 39.37 in

1 km ≈ 0.6214 mi

1 g ≈ 0.0353 oz

1 kg ≈ 2.2046 lb

1 mL ≈ 0.034 oz

1 L ≈ 1.0567 qt

Problem 19:

Change ***33 cm*** to ***inches***. Round your answer to 2 decimal places.

We know that ***1 cm ≈ 0.3937 in***.

All we need to do is multiply both sides of this approximation by 33 to get ***33 cm ≈ 12.99 in***.

Problem 20:

Change ***33 in*** to ***centimeter***. Round your answer to 2 decimal places.

Here we have to use a unit ratio since the only conversion we have is from centimeters to inches. That is, we know that ***1 cm ≈ 0.3937 in***.

The appropriate unit ratio in this case is $\frac{1cm}{0.3937\ in}$ and we find the following:

$$\frac{33\ in}{1}\left(\frac{1cm}{0.3937\ in}\right)=83.82016764cm$$

That is, ***33 in*** is approximately ***83.82 cm***.

Problem 21:

Change ***3.4 lb*** to ***kilograms***. Round your answer to two decimal places.

Again we have to use a unit ratio since the only conversion we have is from kilograms to pounds. That is, we know that ***1 kg ≈ 2.2046 lb***.

Using the appropriate unit ratio, we find the following:

$$\frac{3.4\ lb}{1}\left(\frac{1kg}{2.2046\ lb}\right)=1.542229883\ kg$$

That is, ***3.4 lb*** is approximately ***1.54 kg***.

Problem 22:

Change ***10,000 cc*** to ***ounces***.

We know that ***1 mL = 1 cc***. We further know that

1 mL ≈ 0.034 oz

or

1 cc ≈ 0.034 oz

All we need to do is multiply both sides of this approximation by 10,000 to get
10,000 cc ≈ 340 oz

Problem 23:

Change ***52 feet*** to ***meters***. Round your answer to two decimal places.

First, we'll write the original measure as the fraction $\frac{52\text{ ft}}{1}$

Looking at our conversion charts, we must find some unit ratios.

We know that ***1 m = 39.37 in*** and that ***1 ft = 12 in***.

So let's change feet to inches first and then let's change that to meters.

$$\frac{52\text{ ft}}{1}\left(\frac{12\text{ in}}{1\text{ ft}}\right) = \frac{52(12)\text{ in}}{1(1)}$$
$$= 624\text{ in}$$

Now, we'll multiply the new measure in fraction form with the unit ratio

$\frac{1\text{ m}}{39.37\text{ in}}$ because we know that ***1 m ≈ 39.37 in***

Then

$$\frac{624\text{ in}}{1}\left(\frac{1\text{ m}}{39.37\text{ in}}\right) = \frac{624(1)\text{ m}}{39.37}$$
$$= 15.8496317\text{ m}$$

We find that ***52 feet*** is approximately ***15.85 meters***.

Problem 24:

Change ***2.5 kilograms*** to ***pounds***. Round your answer to two decimal places.

We know that ***1 kg ≈ 2.2046 lb***.

All we need to do is multiply both sides of this approximation by 2.5 to get ***2.5 kg ≈ 5.51 lb***.