



DETAILED SOLUTIONS AND CONCEPTS - OPERATIONS ON FRACTIONS
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YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

ADDITION AND SUBTRACTION

Rule for Adding/Subtracting Fractions and Mixed Numbers:

- Change any whole or mixed numbers to improper fractions.
- Fractions cannot be added/subtracted unless they have the same denominator. If the denominators are not already the same, we must find the **least common denominator**. **
- If necessary, change each fraction to an equivalent fraction using the least common denominator.
- Once all fractions have the same denominator, **we only add/subtract the numerators!!**
- If necessary, reduce the sum to lowest terms and change improper fractions to whole or mixed numbers.

**** The Least Common Denominator (LCD)**

The **Least Common Denominator (LCD)** is the smallest number that is evenly divisible (no remainder) by each of the denominators of the fractions being added. If the denominators are all prime numbers, the LCD is the product of the denominators. If one or more of the denominators is NOT a prime number, the LCD is a product using the largest listing of of each prime number taking into account ALL denominators.

Problem 1:

Find the sum of $\frac{5}{7} + \frac{3}{7}$ reduced to lowest terms.

Here both fractions have the same denominator. All that's left to do is add the numerators.

$$\begin{aligned}\frac{5}{7} + \frac{3}{7} &= \frac{5+3}{7} \\ &= \frac{8}{7} \\ &= 1\frac{1}{7}\end{aligned}$$

Problem 2:

Find the sum of $\frac{2}{5} + \frac{3}{7}$ reduced to lowest terms.

Here both fraction have different denominators. Before we can add them, we will have to make the denominators the same.



Finding the Least Common Denominator:

Only because **5** and **7** are prime numbers, we can find the LCD by multiplying them. That is, the LCD is **7(5) = 35**.



Now we have to change each fraction to an equivalent fraction using the least common denominator (LCD).

$$\frac{2}{5} = \frac{2 \cdot 7}{5 \cdot 7} = \frac{14}{35} \quad \text{and} \quad \frac{3}{7} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{15}{35}$$

and

$$\begin{aligned}\frac{14}{35} + \frac{15}{35} &= \frac{14+15}{35} \\ &= \frac{29}{35}\end{aligned}$$

, which is a proper fraction already reduced to lowest terms!

Problem 3:

Simplify $\frac{5}{6} + \frac{1}{4} - \frac{3}{16}$.

Here we have to add and subtract. The word "simplify" is used often in mathematics and, depending on the situation, requires us to carry out different calculations. Here "simplify" means to add the first two fractions and then subtract the third one from this sum.

All fraction have a different denominator. Before we can simplify, we will have to make the denominators the same.

Finding the Least Common Denominator:

Since **6**, **4**, and **16** are NOT prime numbers, the LCD should not be found by calculating their product.

NOTE: Certainly, **$6(4)(16) = 384$** is a common denominator, but not necessarily the least common denominator. When adding and subtracting fractions we always try to find the least common denominator mainly to keep the calculations manageable. This is especially important when adding/subtracting fractions by hand.

To find the least common denominator (LCD) we will first have to use the concept of **Prime Factorization**. That is, we'll factor each denominator into its prime factors. In our case, we have

$$6 = 2 \cdot 3$$

$$4 = 2 \cdot 2$$

$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

Observe that the factor **3** is listed once. However, the factor **2** is listed once, twice, and four times.

For the LCD, we must always create a product using the largest listing of of each prime number taking into account ALL denominators.

Therefore, we must form a product containing the largest listing of 2 and 3. That is,

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$$

Now we have to change each fraction to an equivalent fraction using the least common denominator (LCD).

$$\frac{5}{6} = \frac{5}{6} \cdot \frac{8}{8} = \frac{40}{48} \quad \text{and} \quad \frac{1}{4} = \frac{1}{4} \cdot \frac{12}{12} = \frac{12}{48} \quad \text{and} \quad \frac{3}{16} = \frac{3}{16} \cdot \frac{3}{3} = \frac{9}{48}$$

and using the *Order of Operation* we get

$$\begin{aligned} \frac{40}{48} + \frac{12}{48} - \frac{9}{48} &= \frac{40 + 12 - 9}{48} \\ &= \frac{52 - 9}{48} \\ &= \frac{43}{48} \end{aligned}$$

This is a proper fraction already reduced to lowest terms!

Problem 4:

A business with a certain revenue must pay $\frac{1}{5}$ for raw materials, $\frac{3}{10}$ for rent and utilities, and $\frac{1}{3}$ for salaries. What fraction of the revenue is left for pure profit? Reduce your answer to lowest terms!

First of all you have to know that "certain revenue" in this case is considered to be **1**. The raw materials, the rent and utilities, and the salaries are a fraction of this "certain revenue" of **1**. Note that we do not need to know in this case how much revenue this business earns!

In order to find the fraction of the revenue that is left over for pure profit, we have to add the existing fractions first and then subtract them from **1**.

That is, $\frac{1}{5} + \frac{3}{10} + \frac{1}{3}$.

Since **10** is not a prime number the LCD should not be found by calculating the product of the denominators. Therefore, we must first use prime factorization.

$$3 = 3$$

$$5 = 5$$

$$10 = 2 \cdot 5$$

Now, we can find the LCD by forming a product containing the largest listing of 2, 3, and 5. That is,

$$2 \cdot 3 \cdot 5 = 30$$

Next, we have to change each fraction to an equivalent fraction using the least common denominator (LCD).

$$\frac{1}{5} = \frac{1}{5} \cdot \frac{6}{6} = \frac{6}{30} \quad \text{and} \quad \frac{3}{10} = \frac{3}{10} \cdot \frac{3}{3} = \frac{9}{30} \quad \text{and} \quad \frac{1}{3} = \frac{1}{3} \cdot \frac{10}{10} = \frac{10}{30}$$

and using the *Order of Operation* we get

$$\begin{aligned} \frac{6}{30} + \frac{9}{30} + \frac{10}{30} &= \frac{6+9+10}{30} \\ &= \frac{25}{30} \\ &= \frac{5}{6} \end{aligned}$$

We find that raw materials, rent and utilities, and salaries take up $\frac{5}{6}$ of the revenue. To find the pure profit, we now need to subtract this from the entire revenue, which is considered to be **1**.

That is, $1 - \frac{5}{6} = \frac{1}{1} - \frac{5}{6}$.

Since one denominator equals **1** and the other one **6**, we can find the least common denominator (LCD) by multiplying them. That is, the LCD is **1(6) = 6**.

Changing the number **1** to a fraction with denominator **6**, we get $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$.

We find that $\frac{1}{6}$ of the revenue is left for pure profit.

Problem 5:

Find the difference of $4 - \frac{3}{8}$ and write as a mixed number reduced to lowest terms.

Here the minuend is a whole number. We will first change it to an improper fraction as follows:

$$4 - \frac{3}{8} = \frac{4}{1} - \frac{3}{8}$$

Since one denominator equals **1** and the other one **8**, we can find the least common denominator (LCD) by multiplying them. That is, the LCD is **1(8) = 8**.

Now we have to change the number **4** to an equivalent fraction using the least common denominator (LCD).

$$\frac{4}{1} = \frac{4 \cdot 8}{1 \cdot 8} = \frac{32}{8}$$

and

$$\begin{aligned}\frac{32}{8} - \frac{3}{8} &= \frac{32 - 3}{8} \\ &= \frac{29}{8} \\ &= 3\frac{5}{8}\end{aligned}$$

The difference is already reduced to lowest terms!

Problem 6:

What number do you have to add to $\frac{10}{29}$ to get $2\frac{17}{29}$? If necessary, express your answer as a mixed number reduced to lowest terms.

We can find this number by subtracting $\frac{10}{29}$ from $2\frac{17}{29}$.

In this case, we only need to subtract the fractions since $\frac{17}{29}$ is greater than $\frac{10}{29}$ and now borrowing from the **2** has to take place. That is, we do not need to change $2\frac{17}{29}$ to a mixed number.

$$\frac{17}{29} - \frac{10}{29} = \frac{17 - 10}{29} = \frac{7}{29}$$

Therefore, we can say that we have to add $2\frac{7}{29}$ to $\frac{10}{29}$ to get $2\frac{17}{29}$.

Problem 7:

Simplify $7\frac{1}{4} + 2\frac{7}{8} - 4\frac{1}{3}$ and write as a mixed number reduced to lowest terms.

Here we are dealing with mixed numbers and different denominator. Therefore, it is best to first change them to improper fractions as follows:

$$7\frac{1}{4} + 2\frac{7}{8} - 4\frac{1}{3} = \frac{29}{4} + \frac{23}{8} - \frac{13}{3}$$

All fractions have a different denominator. Before we can simplify, we will have to make the denominators the same. We do this by using the least common denominator (LCD). This is the smallest number that is evenly divisible (no remainder) by **3**, **4**, and **8**.

4 and **8** are not prime numbers. Therefore, the least common denominator (LCD) is not found by calculating their product.

Following is a listing of the denominators written as a product of prime numbers:

$$3 = 3 \cdot 1$$

$$4 = 2 \cdot 2$$

$$8 = 2 \cdot 2 \cdot 2$$

The factor **3** is listed once. However, the factor **2** is listed twice and three times.

For the LCD, we'll use the largest listing of 2 and the largest listing of 3. That is,

$$2 \cdot 2 \cdot 2 \cdot 3 = 24$$

Now we have to change each fraction to an equivalent fraction using the least common denominator (LCD) and carry out the operations.

$$\begin{aligned} \frac{29}{4} + \frac{23}{8} - \frac{13}{3} &= \frac{29 \cdot 6}{4 \cdot 6} + \frac{23 \cdot 3}{8 \cdot 3} - \frac{13 \cdot 8}{3 \cdot 8} \\ &= \frac{174}{24} + \frac{69}{24} - \frac{104}{24} \\ &= \frac{139}{24} \\ &= 5\frac{19}{24} \end{aligned}$$

This mixed number is already reduced to lowest terms!

MULTIPLICATION

Rule:

- Change any whole or mixed numbers to improper fractions.
- Cross-cancel as much as possible.
- Multiply the numerators AND multiply the denominators.
- If necessary, change improper fractions to whole or mixed numbers.

Problem 8:

Find the product of $\frac{3}{4} \cdot \frac{8}{9}$ reduced to lowest terms.

Please note that instead of using the cross \times or the parentheses () to indicate multiplication, we are using the dot.

Step 2 of the multiplication rule requires us "to cross-cancel as much as possible" before we carry out the multiplication.

Here we notice that **3** and **9**, which are "**across**" the fraction bar from each other, have a factor of **3** in common. Furthermore, **4** and **8**, which are also "**across**" the fraction bar from each other, have a factor of **4** in common. Cross canceling allows us to divide the **3** and **9** by **3** and the **4** and **8** by **4** before we multiply.

$$\begin{array}{r} \cancel{3} \quad \cancel{8} \\ \cancel{4} \quad \cancel{9} \\ \hline 1 \quad 2 \\ 1 \quad 3 \\ \hline \end{array} = \frac{1(2)}{1(3)} = \frac{2}{3}$$

Problem 9:

Find the product of $\frac{3}{4} \cdot \frac{7}{8}$ reduced to lowest terms.

Since cross-canceling is not possible, we simply multiply the numerators and the denominators.

$$\begin{array}{r} \frac{3}{4} \cdot \frac{7}{8} = \frac{3(7)}{4(8)} \\ = \frac{21}{32} \end{array}, \text{ which is a proper fraction reduced to lowest terms}$$

Problem 12:

Find the product of $3\frac{1}{4} \cdot 5$ and write as a mixed number reduced to lowest terms.

Here the multiplicand is a mixed number and the multiplier is a whole number. We will change the whole number to a fraction and the mixed number to an improper fraction.

$$\frac{13}{4} \cdot \frac{5}{1}$$

In this case cross canceling is not possible because all numbers only have a factor of 1 in common, To find the product all we have to do is multiply the numerators and the denominators to get

$$\begin{aligned} \frac{13}{4} \cdot \frac{5}{1} &= \frac{13(5)}{4(1)} \\ &= \frac{65}{4} \\ &= 16\frac{1}{4} \end{aligned}$$

This is a mixed number already reduced to lowest terms!

Problem 13:

Find the product of $13 \cdot 3\frac{4}{7}$ and express as a mixed number reduced to lowest terms.

Here the multiplicand is a whole number and the multiplier is a mixed number. We will change the whole number to a fraction and the mixed number to an improper fraction.

$$\frac{13}{1} \cdot \frac{25}{7}$$

In this case cross canceling is not possible because all numbers only have a factor of 1 in common, To find the product all we have to do is multiply the numerators and the denominators to get

$$\frac{325}{7} = 46\frac{3}{7}$$

This is a mixed number already reduced to lowest terms!

Problem 14:

What number is $\frac{3}{10}$ of **150**?

Whenever you see the word "of" in conjunction with fraction you have to think MULTIPLICATION !!!

That is, we can write the expression $\frac{3}{10}$ of **150** as $\frac{3}{10} \cdot 150$.

Then we find $\frac{3}{10} \cdot 150 = \frac{3}{10} \cdot \frac{150}{1}$

and after cross canceling we get $\frac{3}{1} \cdot \frac{15}{1}$, which equals $\frac{45}{1}$ or **45**.

DIVISION

Rule:

- Change any whole or mixed numbers to improper fractions.
- Convert the division to an equivalent multiplication problem using the **reciprocal** ** of the divisor as the multiplier.
- Then multiply according to the **rule for multiplying fractions and mixed numbers**.

** Interchanging the numerator and denominator of a fraction results in a fraction that is called the **reciprocal** of the original fraction. When a number is multiplied by its reciprocal, the product equals **1**.

Problem 15:

Find the quotient of $\frac{5}{8} \div \frac{3}{4}$ reduced to lowest terms.

In order to carry out this division, we will convert it to an equivalent multiplication problem using the reciprocal of the divisor as the multiplier.

That is, $\frac{3}{4}$ becomes $\frac{4}{3}$.

$$\frac{5 \overset{1}{\cancel{4}}}{\overset{2}{\cancel{8}} 3} = \frac{5(1)}{2(3)}$$

$$= \frac{5}{6}$$

This is a proper fraction already reduced to lowest terms!

Problem 16:

Find the quotient of $21 \div 3\frac{5}{8}$ and write as a mixed number reduced to lowest terms.

Here the multiplicand is a whole number and the multiplier is a mixed number. We will first change them to improper fractions.

$$\frac{21}{1} \div \frac{29}{8}$$

In order to carry out this division, we will convert it to an equivalent multiplication problem using the reciprocal of the divisor as the multiplier.

$$\frac{21}{1} \cdot \frac{8}{29} = \frac{21(8)}{1(29)}$$

$$= \frac{168}{29}$$

$$= 5\frac{23}{29}$$

This is a mixed number already reduced to lowest terms!

Problem 17:

Find the quotient of $6\frac{7}{9} \div 5$ and write as a mixed number reduced to lowest terms.

Here the multiplicand is a mixed number and the multiplier is a whole number. We will first change them to improper fractions.

$$\frac{61}{9} \div \frac{5}{1}$$

In order to carry out this division, we will convert it to an equivalent multiplication problem using the reciprocal of the divisor as the multiplier.

$$\frac{61}{9} \cdot \frac{1}{5} = \frac{61(1)}{9(5)}$$

$$= \frac{61}{45}$$

$$= 1 \frac{16}{45}$$

This is a mixed number already reduced to lowest terms!

Problem 18:

Find the quotient of $\frac{4}{\frac{1}{2} + \frac{3}{8} + \frac{5}{16}}$ and write as a mixed number reduced to lowest terms.

In this case, we are dividing the number 4 by the sum of three fractions. This is called a **complex fraction**. The first thing we need to do is to find this sum!

All fraction have a different denominator. Therefore, before we can add we will have to make the denominators the same.

Finding the Least Common Denominator:

Since **8** and **16** are not prime numbers the LCD should not be found by calculating their product.

To find the least common denominator (LCD) we will first have to use the concept of **Prime Factorization**. That is, we'll factor each denominator into its prime factors

$$2 = 2$$

$$8 = 2 \cdot 2 \cdot 2$$

$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

Observe that the factor **2** is listed once, three times, and four times.

For the LCD, we must always create a product using the largest listing of of each prime number taking into account ALL denominators.

In our case, we form a product containing the largest listing of 2. That is,

$$2 \cdot 2 \cdot 2 \cdot 2 = 16$$

Now we have to change two fraction to an equivalent fraction using the least common denominator (LCD).

$$\frac{1}{2} = \frac{1}{2} \cdot \frac{8}{8} = \frac{8}{16} \quad \text{and} \quad \frac{3}{8} = \frac{3}{8} \cdot \frac{2}{2} = \frac{6}{16}$$

and using the *Order of Operation* we get

$$\begin{aligned} \frac{8}{16} + \frac{6}{16} + \frac{5}{16} &= \frac{8+6+5}{16} \\ &= \frac{19}{16} \end{aligned}$$

Now, we can write $\frac{1}{2} + \frac{3}{8} + \frac{5}{16}$ as $\frac{4}{16}$.

This is equal to $4 \div \frac{19}{16}$ and we find the quotient by changing the divisor to its reciprocal.

$$\begin{aligned} 4 \cdot \frac{16}{19} &= \frac{4}{1} \cdot \frac{16}{19} \\ &= \frac{64}{19} \\ &= 3\frac{7}{19} \end{aligned}$$

This is a mixed number already reduced to lowest terms!