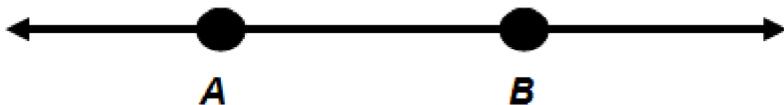




DETAILED SOLUTIONS AND CONCEPTS - LINES AND ANGLES
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Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

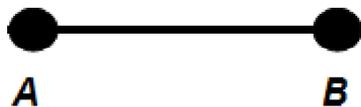
Definition of a Line

A line extends indefinitely in both directions indicated by arrows. A line can be identified by naming any two points on the line, for example, **A** and **B**. The notation for a line that extends through points **A** and **B** is \overleftrightarrow{AB} (in alphabetic order).



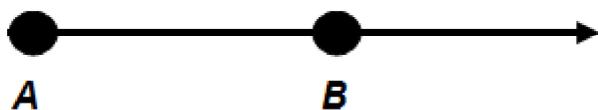
Definition of a Line Segment

A line segment starts and stops at distinct points called endpoints. A line segment can be identified by its endpoints, for example, **A** and **B**. The notation for a line segment that includes the points **A** and **B** is \overline{AB} (in alphabetic order).



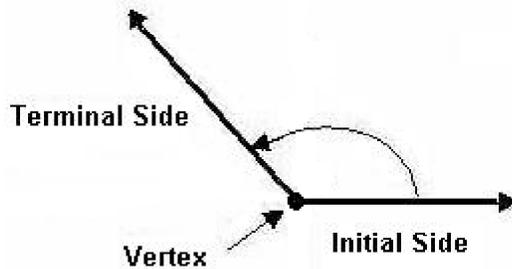
Definition of a Ray

A ray consists of a point on a line and all subsequent points on one side of the point. The point from which the ray originates is called the endpoint. A ray is named by its endpoint and any other point on the ray, for example, **A** and **B**. The notation for a ray that includes the points **A** and **B** is \overrightarrow{AB} (in alphabetic order).



Definition of an Angle

An angle is determined by rotating a ray about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side** of the angle. The point where the initial and the terminal side meet is called the **vertex** of the angle.



Angle Measure

Most commonly, angles are measured in degrees. This is indicated by the degree sign $^{\circ}$ next to a number. For example, 45° .

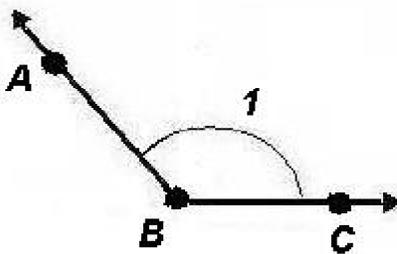
Degrees can further be divided into minutes (') and seconds ("). That is,

$1^{\circ} = 60'$ (minutes) **using the apostrophe on the computer keyboard**

$1' = 60''$ (seconds) **using the quotation mark on the keyboard**

Naming Angles

Angles can be named in several ways. In this course we are going to use three different ways. We'll show them by using the following picture:



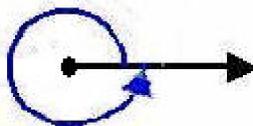
- We can place a number or letter in between the two rays, say **1** or the common Greek letter θ (theta) and then name the angle $\angle 1$ or $\angle \theta$ using the symbol \angle for angle. We pronounce this "angle 1" or "angle theta."
- We can also use the letters of points on the rays together with the vertex point. That is, $\angle ABC$ or $\angle CBA$, either way, as long as the letter for the vertex point is in the middle.
- Finally, we can use the letter for the vertex point alone as long as it is perfectly clear which angle is designated by this letter. In the picture above, it is quite clear which angle we mean when we say $\angle B$.

Zero-Degree Angle

One ray indicates the initial and the terminal side.



360° Degree Angle



NOTE: Unlike with the 0° angle, there is an arrow indicating that the terminal side moved through a rotation of 360° .

Right Angles

Angles whose measure is exactly 90° .



Straight Angles

Angles whose measure is exactly 180° .



Acute Angles

Angles whose measure is greater than 0° but less than 90° .

Obtuse Angles

Angles whose measure is greater than 90° but less than 180° .

Complementary Angles

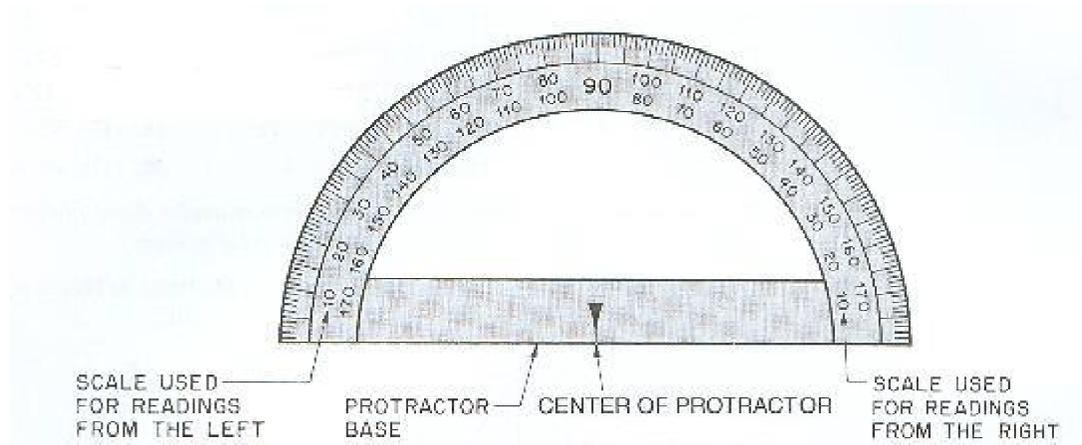
Two angles are called *complementary* when their sum is 90° .

Supplementary Angles

Two angles are called *supplementary* when their sum is 180° .

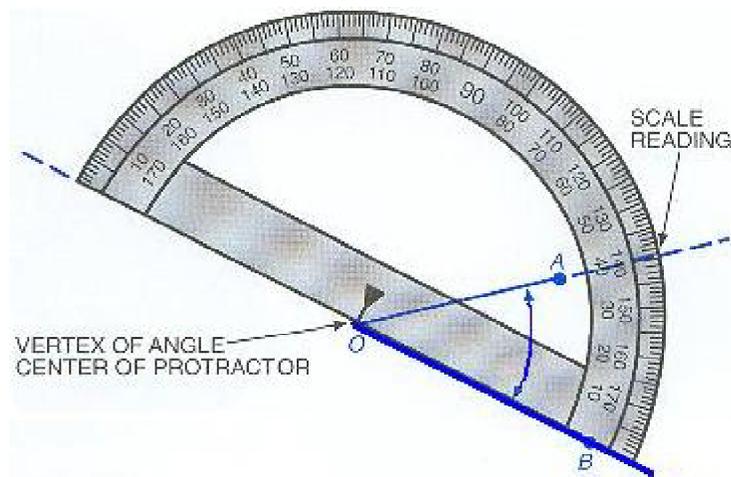
Measuring an Angle

We use a semicircular protractor to measure angles. Below is a picture of one type.



Measuring Procedure:

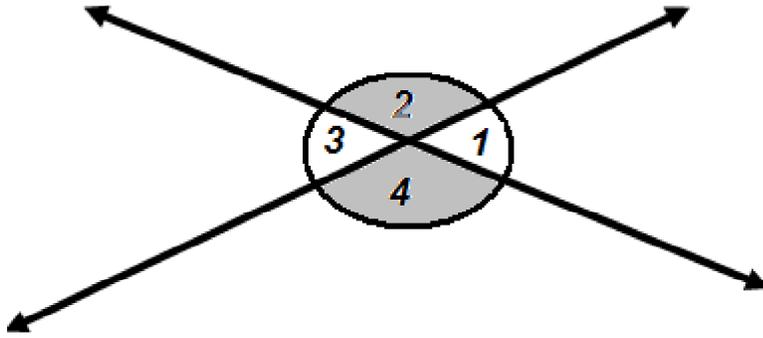
- Place the base of the protractor along one side of the angle with the point on your protractor that indicates its "center" on the vertex of the angle.
- Please note that the "center" of the protractor is somewhere along or close to the base of the protractor. Be very careful! Not all protractors have user-friendly centers.
- Depending on the location of the other side of the angle, either choose the scale that has the zero-degree reading on the right side of the protractor or on the left side. Read the measurement where the side crosses the scale of the protractor.



The measure of angle $\angle O$ is 40° .

Relationships Among Angles Formed by Intersecting Lines

Below is the picture of two intersecting lines.



We can say the following about the angles form by these lines:

The measures of opposite angles are equal. In the picture above, $m\angle 1 = m\angle 3$. Likewise, $m\angle 2 = m\angle 4$.

NOTE: When we discuss the measure of an angle, traditionally the letter m is placed before the angle symbol. For example, $m\angle ABC = m\angle DEF$ tells us that the measures of angles ABC and DEF are the same.

The sum of the measures of adjacent angles equal 180° . In the picture above, $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 3 + m\angle 4 = 180^\circ$, $m\angle 1 + m\angle 4 = 180^\circ$ and $m\angle 2 + m\angle 3 = 180^\circ$

Problem 1:

Change $45^\circ 14' 39''$ (45 degrees and 14 minutes and 39 seconds) to decimal degree form. Round to two decimal places.

Calculate $45 + \frac{14}{60} + \frac{39}{3600}$. Type the entire calculation into your calculator. Do not round until you have the final answer: $45^\circ 14' 39'' \approx 45.24^\circ$

Problem 2:

Change $34' 25''$ (34 minutes and 25 seconds) to decimal degree form. Round to two decimal places.

Calculate $\frac{34}{60} + \frac{25}{3600}$. Type the entire calculation into your calculator. Do not round until you have the final answer: $34' 25'' \approx 0.57^\circ$

Problem 3:

Change $55^{\circ}29''$ (55 degrees and 29 seconds) to decimal degree form. Round to three decimal places.

Calculate $55 + \frac{29}{3600}$. Type the entire calculation into your calculator. Do not round until you have the final answer: $55^{\circ}29'' \approx 55.008^{\circ}$

Problem 4:

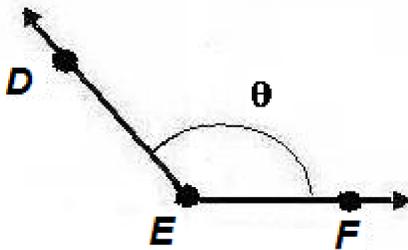
Change 84.78° to degrees, minutes, and seconds rounded to whole numbers.

Take $.78$ away from 84.78 and calculate the minutes as follows: $.78(60)' = 46.8'$

Take $.8$ away from 46.8 and calculate the seconds as follows: $.8(60)'' = 48''$

Thus, $84.78^{\circ} = 84^{\circ}46'48''$

Problem 5:



a. Name the angle in two different ways using three capital letters.

$\angle DEF$ or $\angle FED$ The vertex point is in the middle!

b. Name the angle using one capital letter.

$\angle E$ using the letter of the vertex point!

c. Name the angle using the Greek letter θ (theta) between the two rays.

$\angle \theta$

Problem 6:

Classify the angles as right, straight, acute, or obtuse:

- 38° - acute angle because the measure is greater than 0° but less than 90°
- 95° - obtuse angle because the measure is greater than 90° but less than 180°
- 90° - right angle
- 153° - obtuse angle because the measure is greater than 90° but less than 180°
- 10° - acute angle because the measure is greater than 0° but less than 90°
- 180° - straight angle

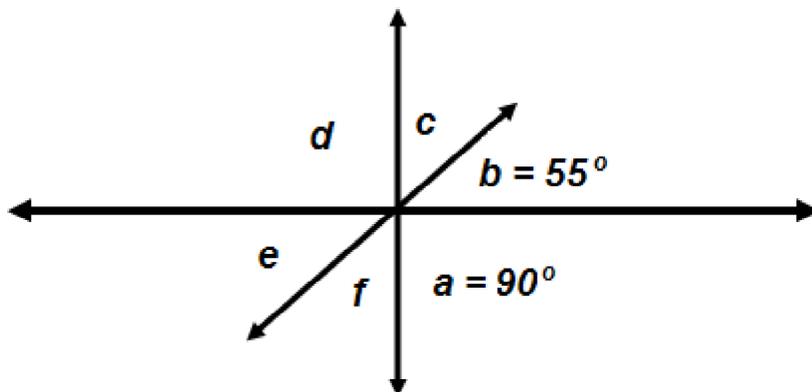
Problem 7:

Tell whether the angle pairs are complementary, supplementary, or neither.

- 42° , 80° - sum equals 122° , which is neither 90° nor 180°
- 17° , 73° - sum equals 90° , therefore, angles are complementary
- 38° , 142° - sum equals 180° , therefore, angles are supplementary
- 52° , 48° - sum equals 100° , which is neither 90° nor 180°
- 60° , 30° - sum equals 90° , therefore, angles are complementary
- 110° , 70° - sum equals 180° , therefore, angles are supplementary

Problem 8:

In the figure shown below, you are given the measure of two angles. Find the measure of the remaining angles.



Angles **a** and **d** are opposite angles. Therefore, $m\angle a$ equals $m\angle d$. The measure of $\angle d$ must be 90° . This indicates that the angle **b + c** must have a measure of 90° because the angles **a** and **b + c** are adjacent angles who are supplementary!

Angles **b** and **c** are complementary angles. Therefore, $m\angle b + m\angle c = 90^\circ$. The measure of $\angle c$ must be $90^\circ - 55^\circ = 35^\circ$.

Angles **b** and **e** are opposite angles. Therefore, $m\angle b$ equals $m\angle e$. The measure of $\angle e$ must be 55° .

Angles **c** and **f** are opposite angles. Therefore, $m\angle c$ equals $m\angle f$. The measure of $\angle f$ must be 35° .