



DETAILED SOLUTIONS AND CONCEPTS - POLYNOMIAL EXPRESSIONS
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PLEASE NOTE THAT YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Polynomial Expression

The sum or difference of terms in which the exponents of the variables are nonnegative integers (positive integers plus 0) and there are no variables in a denominator.

Term - Mathematical expressions that are single variables, constants, products, or quotients. They are separated from each other by plus or minus signs.

Variables - Some equations contain unknown numbers, which are usually represented by letters of the alphabet. In mathematics we most often use the letters **x** and **y**. These letters are called *variables* or *unknowns*.

Problem 1:

Decide whether or not the following algebraic expressions are polynomials. Answer Yes or No!

a. $5x^3 + 2x + 5$

Yes

b. $6x + \frac{5}{x^2}$

No, because there is one term that has a variable in the denominator.

c. $\frac{4}{3}x^2 - \frac{1}{2}x$

Yes

d. **5**

Yes, because **5** is equivalent to $5x^0$ and the power 0 is part of the nonnegative integers. Remember that x^0 equals **1**!

e. **$3x - 1$**

Yes

f. **$-4x^3 + 3x^{-2}$**

No, because one variable has a power that is a negative integer.

Special Polynomial Expressions

A **monomial** is a polynomial containing one term. For example, **5**, **$-3y$** , **$6abc$** , or **$9x^2y^3$**

A **binomial** is a polynomial containing two terms. For example, **$x + 5$** or **$-3x^2 - 7x$**

A **trinomial** is a polynomial containing three terms. For example, **$5 + a - b$** or **$x^2 - 2x - 8$**

Adding and Subtracting Polynomial Expressions

Since a polynomial expression contains *terms*, let's review the rules on adding and subtracting *terms*.

- We can only add and subtract *like terms*. Specifically, we add and subtract their *coefficients* only.

For example, $2x^2 + x^2$ can be simplified to $3x^2$. **Remember, when you don't see a coefficient it is assumed to be 1.**

However, $2x^2 + x$ cannot be further simplified because the powers of the variable are not the same.

Terms of polynomials containing the same variable base are most often arranged in descending order of their exponent.

For example, it is customary to write $-3a^5 - a^2$ instead of $-a^2 - 3a^5$. Although, an answer is not incorrect if the variables are not arranged in descending order of their exponent.

Problem 2:

Add $6x^3 + 7x^3$

$6x^3 + 7x^3 = (6 + 7)x^3 = 13x^3$ Remember, only the coefficients are added or subtracted!

Problem 3:

Subtract $a^5 - 3a^5$.

$a^5 - 3a^5 = (1 - 3)a^5 = -2a^5$ Remember, when you don't see a coefficient it is assumed to be 1.

Problem 4:

Combine $a^5 + 5a^2 - 4a^5 - 6a^2$ and write the terms in descending order of their exponents.

NOTE: The word "combine" in mathematics often indicates that you are supposed to add or subtract!

Here we need to group the terms containing like variables raised to like powers. You may do this mentally as long as you don't get confused!

$$\begin{aligned} a^5 + 5a^2 - 4a^5 - 6a^2 &= a^5 - 4a^5 + 5a^2 - 6a^2 \\ &= -3a^5 - a^2 \end{aligned}$$

Note 1: The exponent 5 is larger than the exponent 2. The order is descending!

Note 2: A coefficient of **1** and **-1** is implied and is usually not written! That is why we did not write **-1a²**.

Problem 5:

Combine $a + a + a^2 - a^3$ and write the terms in descending order of their exponents.

$$a + a + a^2 - a^3 = -a^3 + a^2 + 2a$$

Note: The power 3 is larger than the power 2, which is larger than the power 1. The order is descending! Remember that $a = a^1$!

Problem 6:

Combine $x^3 + 5x - (3x^3 + 5x)$ and write the terms in descending order of their exponents.

Here we have to use the *Distributive Property of Multiplication* first according to the *Order of Operation*. That is, we have to multiply the terms in the parentheses with **-1** as follows:

$$x^3 + 5x - (3x^3 + 5x) = x^3 + 5x - 3x^3 - 5x$$

$$= -2x^3$$

Note that the x-term was "subtracted out."

Problem 7:

Combine $9 + 3x + 5 + x^2 + x^2$ and write the terms in descending order of their exponents.

$$9 + 3x + 5 + x^2 + x^2 = 2x^2 + 3x + 14$$

Note that the term 14 is assumed to be $14x^0$ and we know that $x^0 = 1$. Then the power 2 is larger than the power 1, which is larger than the power 0. The order is descending!!

Multiplying Polynomial Expressions

1. Finding Products having one Monomial and one Binomial Factor.

Problem 8:

Multiply $3(2a^2 + 3ab - 4b^2)$.

Here we need to use the *Distributive Property of Multiplication*. It states that if you have a product of two factors, where one factor consists of an algebraic expression enclosed in parentheses, you can multiply each term of the algebraic expression with the other factor.

$$3(2a^2 + 3ab - 4b^2) = 3(2a^2) + 3(3ab) + 3(-4b^2)$$

$$= 6a^2 + 9ab - 12b^2$$

Please note the following:

The expression $3(2a^2) + 3(3ab) + 3(-4b^2)$ could have also been written as follows. Please observe the last term!

$3(2a^2) + 3(3ab) - 3(4b^2)$ This is probably more customary than the first representation!

Problem 9:

Multiply $3x(7 + 3x^2)$ and write the terms in descending order of their exponents.

Again we are using the *Distributive Property of Multiplication*.

$$3x(7 + 3x^2) = 3x(7) + 3x(3x^2)$$

$$= 21x + 9x^3$$

and writing the terms in descending order of their exponents we get $9x^3 + 21x$.

Problem 10:

Multiply $3x(7 - 3x^2)$ and write the terms in descending order of their exponents.

$$\begin{aligned}3x(7 - 3x^2) &= 3x(7) - 3x(3x^2) \text{ It's okay to switch the signs in the second product!} \\ &= 21x - 9x^3\end{aligned}$$

and writing the terms in descending order of their exponents we get $-9x^3 + 21x$.

Problem 11:

Multiply $-3x(7 + 3x^2)$ and write the terms in descending order of their exponents.

$$\begin{aligned}-3x(7 + 3x^2) &= -3x(7) - 3x(3x^2) \\ &= -21x - 9x^3\end{aligned}$$

and writing the terms in descending order of their exponents we get $-9x^3 - 21x$.

Problem 12:

Multiply $-3x(7 - 3x^2)$ and write the terms in descending order of their exponents.

$$\begin{aligned}-3x(7 - 3x^2) &= -3x(7) - 3x(-3x^2) \\ &= -21x + 9x^3\end{aligned}$$

and writing the terms in descending order of their exponents we get $9x^3 - 21x$.

2. Finding Products having two Binomial Factors

Here we use an extension of the *Distributive Property of Multiplication*, The extension simply says that you can multiply, in turn, each term of the first factor with the entire second factor.

Problem 13:

Multiply $(x + 4)(x - 2)$. Combine like terms, if necessary and write the terms in descending order of their exponents.

$(x + 4)(x - 2) = x(x - 2) + 4(x - 2)$ Note that each term of the first factor is being multiplied with the entire second factor.

Now, we have two products, which we will simplify using the *Distributive Property* again. However, we won't write down the individual multiplications, but rather do them mentally. Lastly, we will combine like terms!

$$\begin{aligned}(x + 4)(x - 2) &= x(x - 2) + 4(x - 2) \\ &= x^2 - 2x + 4x - 8 \\ &= x^2 + 2x - 8\end{aligned}$$

Problem 14:

Multiply $(2x - 5x^2)(3x^3 + 9)$ and write the terms in descending order of their exponents.

$$(2x - 5x^2)(3x^3 + 9) = 2x(3x^3 + 9) - 5x^2(3x^3 + 9)$$

We again have two products, which we will again simplify using the *Distributive Property* one more time. However, in this case we will write down the individual multiplications because they involve more complicated terms.

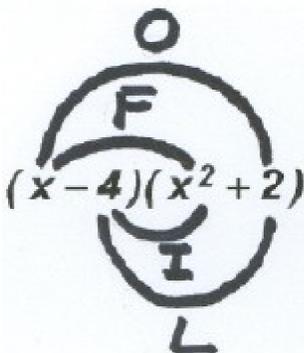
$$\begin{aligned}(2x - 5x^2)(3x^3 + 9) &= 2x(3x^3 + 9) - 5x^2(3x^3 + 9) \\ &= 2x(3x^3) + 2x(9) - 5x^2(3x^3) - 5x^2(9) \\ &= 6x^4 + 18x - 15x^5 - 45x^2\end{aligned}$$

and writing the terms in descending order of their exponents we get $-15x^5 + 6x^4 - 45x^2 + 18x$.

Problem 15:

Use the **FOIL Method** to multiply $(x - 4)(x^2 + 2)$. Combine like terms, if necessary and write the terms in descending order of their exponents.

FOIL is a shortcut method for multiplying two binomials. It is an acronym for **F**irst **O**uter **I**nner **L**ast, which refers to the order in which the terms of the two binomial are multiplied. Please look at the following picture:



$$(x - 4)(x^2 + 2) = x(x^2) + 2x - 4x^2 - 4(2)$$

$$= x^3 + 2x - 4x^2 - 8$$

Please note that for the Outer Product we wrote **2x** instead **x(2)**. It is customary procedure to write the numeric factor before the variable factor!!!

The last thing we want to do is to write the product in descending order of their exponents. That is,

$$(x - 4)(x^2 + 2) = x^3 - 4x^2 + 2x - 8$$

Problem 16:

Use the FOIL Method to multiply **(2x - 3)(x - 2)**. Combine like terms, if necessary and write the terms in descending order of their exponents.

$$\begin{array}{cccc} & F & O & I & L \\ (2x - 3)(x - 2) & = & 2x(x) & + & 2x(-2) & - & 3x & - & 3(-2) \\ & = & 2x^2 & - & 4x & - & 3x & + & 6 \\ & = & 2x^2 & - & 7x & + & 6 \end{array}$$

Problem 17:

Use the FOIL Method to multiply **(x + 3)(x - 2)**. Combine like terms, if necessary and write the terms in descending order of their exponents.

$$\begin{array}{cccc} & F & O & I & L \\ (x + 3)(x - 2) & = & x^2 & - & 2x & + & 3x & - & 6 \\ & = & x^2 & + & x & - & 6 \end{array}$$

Note: A coefficient of 1 and -1 is implied and is usually not written! That is why we did not write 1x.

Problem 18:

Use the FOIL Method to multiply **(kx + 3)(x - t)**.

$$\begin{array}{cccc} & F & O & I & L \\ (kx + 3)(x - t) & = & kx^2 & - & ktx & + & 3x & - & 3t \end{array}$$

Please notice that in this case we are not able to combine the two middle terms like we did in Problem 17 by stating that **-2x + 3x = x**. Here we have **-ktx + 3x**.

Problem 19:

Simplify $(5 - x)^2$. Combine like terms, if necessary and write the terms in descending order of their exponents.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we will be asked to "simplify" instead of to squaring the term.

Please note that $(5 - x)^2 \neq 5^2 - x^2$. However, we do know that $(5 - x)^2 = (5 - x)(5 - x)$. Now we can use FOIL to simplify this product.

$$\begin{aligned}(5 - x)^2 &= (5 - x)(5 - x) \\ &\quad \text{F O I L} \\ &= 25 - 5x - 5x + x^2 \\ &= x^2 - 10x + 25\end{aligned}$$

Please note that for the inner product we wrote $-5x$ instead of $-x(5)$. Also, observe that the simplified product was written with terms arranged in descending order of the exponent.

Problem 20:

Simplify $(5 + x)^2$. Combine like terms, if necessary and write the terms in descending order of their exponents.

Please note that $(5 + x)^2 \neq 5^2 + x^2$. However, we do know that $(5 + x)^2 = (5 + x)(5 + x)$. Now we can use FOIL to simplify this product.

$$\begin{aligned}(5 + x)^2 &= (5 + x)(5 + x) \\ &\quad \text{F O I L} \\ &= 25 + 5x + 5x + x^2 \\ &= x^2 + 10x + 25\end{aligned}$$

Please note that for the inner product we wrote $5x$ instead of $x(5)$. Also, observe that the simplified product was written with terms arranged in descending order of the exponent.

Dividing Polynomial Expressions by Monomials

Please note that when a polynomial expression is divided by a monomial each and every term in the numerator is divided by the denominator.

Problem 21:

Divide $\frac{8x^3 - 4x^2}{2x^2}$ and write the reduced terms in descending order of their exponents.

We must divide each and every term in the numerator by the denominator and then use the *Laws of Exponents* to simplify! The laws of exponents should be done mentally, whenever possible, otherwise the calculations become too unwieldy.

$$\frac{8x^3 - 4x^2}{2x^2} = \frac{8x^3}{2x^2} - \frac{4x^2}{2x^2} = 4x - 2$$

Note that we used the *Law of Exponents* $\frac{a^m}{a^n} = a^{m-n}$ to simplify each term.

Please note, that you CANNOT do the following:

$$\frac{8x^3 - 4x^2}{2x^2} \stackrel{21}{=} 8x^3 - 2$$

THIS IS NOT CORRECT !!! Each and every term in the numerator must be divided by the denominator.

Problem 22:

Divide $\frac{18x^3y^2 + 9x^4y^3 - 24x^6y^3z^2}{-3xy^2}$ and reduce to lowest terms if possible.

Again, we must divide each and every term in the numerator by the denominator and then use the *laws of exponents* to simplify!

$$\frac{18x^3y^2 + 9x^4y^3 - 24x^6y^3z^2}{-3xy^2} = \frac{18x^3y^2}{-3xy^2} + \frac{9x^4y^3}{-3xy^2} - \frac{24x^6y^3z^2}{-3xy^2} = -6x^2 - 3x^3y + 8x^5yz^2$$

Note that we used the *Law of Exponents* $\frac{a^m}{a^n} = a^{m-n}$ to simplify each variable in each term. Incidentally, in this case it would be pointless to write the terms in descending order of their exponents.

Problem 23:

$$\frac{3a^4b^5 - 9ab}{9ab}$$

Divide $9ab$ and write the terms in descending order of their exponent.

We must divide each and every term in the numerator by the denominator and then use the *laws of exponents* to simplify! The laws of exponents should be done mentally, whenever possible, otherwise the calculations become too unwieldy.

$$\begin{aligned} \frac{3a^4b^5 - 9ab}{9ab} &= \frac{3a^4b^5}{9ab} - \frac{9ab}{9ab} \\ &= \frac{a^3b^4}{3} - 1 \end{aligned}$$

Note that we used the *Law of Exponents*

$$\frac{a^m}{a^n} = a^{m-n}$$

to simplify each term.

We can also express the answer as $\frac{1}{3}a^3b^4 - 1$.

Problem 24:

Multiply $\left(\frac{6a}{7b}\right)\left(\frac{b^3}{2a}\right)$ and reduce to lowest term, if possible.

$$\left(\frac{6a}{7b}\right)\left(\frac{b^3}{2a}\right) = \frac{6ab^3}{14ab}$$

Now we can reduce as follows using the *Law of Exponents* $\frac{a^m}{a^n} = a^{m-n}$.

$$\begin{aligned} \left(\frac{6a}{7b}\right)\left(\frac{b^3}{2a}\right) &= \frac{\overset{3}{\cancel{6}} \overset{1}{\cancel{a}} b^3}{\underset{7}{\cancel{14}} \underset{1}{\cancel{a}} b} \\ &= \frac{3b^2}{7} \end{aligned}$$

Please note that $\frac{a}{a} = 1$ and $\frac{b^3}{b} = b^2$. Furthermore, we can write $\frac{3b^2}{7}$ as $\frac{3}{7}b^2$.