DETAILED SOLUTIONS AND CONCEPTS - SYSTEMS OF LINEAR EQUATIONS

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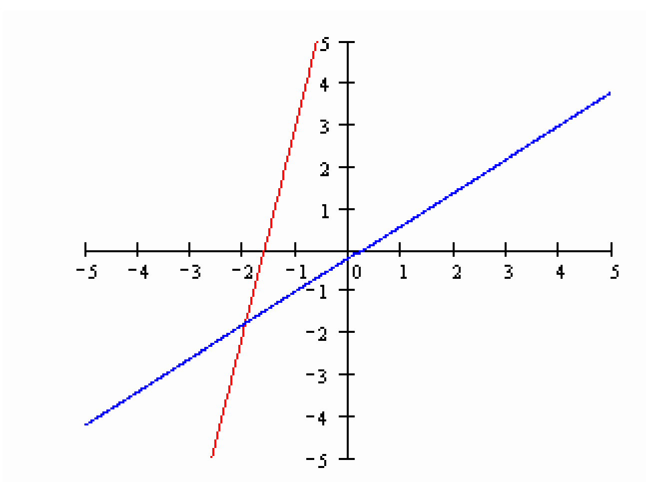
PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Systems of Linear Equations

You already studied the linear equation in two variables $Ax + By = C$ whose graph is a straight line. Many applied problems are modeled by two or more linear equations. When this happens, we talk about a *System of Linear Equations*.

Systems of Linear Equations can be solved graphically, or by using the *Substitution Method* or the *Addition Method Method*.

The following is a pictorial representation of the system consisting of the linear equations $-5x + y = 8$ and $-4x + 5y = -1$. The point of intersection is considered the solution of the system.



Solving a *System of Linear Equations* graphically often does not give use the correct solution. For example, in the picture above we might believe that the solution is $(-2,-2)$.

Actually, the solution is $(-\frac{41}{21},-\frac{37}{21})$ as you will see later, but this is difficult to see on a graph. Therefore, we will consider two methods for solving linear systems that do NOT depend on finding solutions visually.

Strategy for Solving Systems of Equations by the Substitution Method

Step 1: Solve any one of the equations for one variable in terms of the other. If one of the equations is already in this form, you can skip this step.

Step 2: Substitute the expression found in Step 1 into the other equation. You should now have an equation in one variable. Find its value.

> Please note that ONLY at the point of intersection two equations are equal to each other. By setting the x- or y-value of one equation equal to the x- or y-value of the other equation, we are in effect finding the point of intersection.

Step 3: To find the value of the second variable, back-substitute the value of the variable found in Step 2 into one of the original equations.

Step 4: Form an ordered pair with the values found in Step 3 and Step 4. This is the solution to your system of equations.

Strategy for Solving Systems of Equations by the Addition Method

Step 1: If necessary, rewrite both equations in the same form so that the variables and the constants match up when one equation is beneath the other.

Step 2: If necessary, multiply either equation or both equations by appropriate numbers so that the coefficient of either x or y will be opposite in sign giving a sum of 0.

Step 3: Write the equations one below the other, draw a horizontal line, then add each of their terms. The sum should be an equation in one variable. Find its value.

Step 4: To find the value of the second variable, back-substitute the value of the variable found in Step 3 into one of the original equations.

Step 5: Form an ordered pair with the values found in Step 4 and Step 5. This is the solution to your system of equations.

NOTE: Solving *Systems of Linear Equations* with the *Substitution Method* is often quicker than when using the *Addition Method*. However, the *Substitution Method* is most useful if one of the given equations has a variable with coefficient 1 or -1. Such a variable can easily be isolated without introducing fractions. As soon as isolating either of the two variables in both equations produces ALL fractions, the *Addition Method* becomes a better choice.

Problem 1:

Solve the following system:

$$-5x + y = 8$$

$$-4x + 5y = -1$$

Let's use both the *Substitution Method* and the *Addition Method*.

Substitution Method:

Step 1:

Solve any one of the equations for one variable in terms of the other. By solving $-5x + y = 8$ for y, we find

$$y = 5x + 8$$

Step 2:

Next, we back-substitute into the equation $-4x + 5y = -1$ as follows

$$-4x + 5(5x + 8) = -1$$

$$-4x + 25x + 40 = -1$$

$$21x = -41$$

$$x = -\tfrac{41}{21}$$

Step 3:

Now, we have to find the y-coordinate of the point of intersection of the two lines. Back-substituting into the equation $-5x + y = 8$, we get

$$-5(-\tfrac{41}{21}) + y = 8$$

$$\tfrac{205}{21} + y = 8$$

$$y = -\tfrac{37}{21}$$

Therefore, the solution to the linear system of equations or the point of intersection of the two lines is $(-\tfrac{41}{21}, -\tfrac{37}{21})$.

Addition Method:

$$-5x + y = 8$$

$$-4x + 5y = -1$$

Step 1:

The equations are already rewritten in the same form so that the variables and the constants match up when one equation is beneath the other.

Step 2:

We multiply both sides of the first equation by ***-5*** so that the coefficient of ***y*** in both equations will be opposite in sign giving a sum of ***0***.

$$-5(-5x + y) = -5(8)$$

$$25x - 5y = -40$$

Step 3:

Now, we will write the "new" first equation below the second equation, draw a horizontal line, then add each of their terms.

$$\begin{array}{rrcr} & 25x - 5y & = & -40 \\ & -4x + 5y & = & -1 \\ + & \hline 21x & = & -41 \\ & x & = & -\frac{41}{21} \end{array}$$

The x-coordinate of the point of intersection.

Step 4:

To find the y-coordinate of the point of intersection of the two lines, we back-substitute the value found for ***x*** into the equation $-5x + y = 8$ to get

$$-5\left(-\frac{41}{21}\right) + y = 8$$

$$\frac{205}{21} + y = 8$$

$$y = -\frac{37}{21}$$

The solution to the system of linear equations or their point of intersection is $\left(-\frac{41}{21}, -\frac{37}{21}\right)$**.**

Problem 2:

Solve the following system:

$$3x - 3y = -2$$

$$x - y = 5$$

We will use the *Substitution Method* because both variables in the second equation have a coefficient of 1.

Solve the second equation for one variable in terms of the other. It does not matter which one you solve for!

By solving the equation $x - y = 5$ for x, we find $x = y + 5$.

Back-substituting into the equation $3x - 3y = -2$, we get

$$3(y + 5) - 3y = -2$$

$$3y + 15 - 3y = -2$$

$15 = -2$, which is, of course, impossible.

In this case, we can conclude that the *System of Linear Equations* has **NO** solutions. This indicates that the two lines are parallel to each other. Remember that parallel lines have the same slope!

Problem 3:

Solve the following system:

$$y = 2x - 3$$
$$y = 4x + 5$$

We will use the *Substitution Method* because both equations have variables with a coefficient of 1.

In this case, both equations are already solved for y. This is a perfect case for the *Substitution Method*. We just have to substitute the right side of the first equation for y in the second equation. That is,

$$2x - 3 = 4x + 5$$

$$-2x = 8$$

$$x = -4$$

Solving for the y-coordinate using the equation $y = 2x - 3$, we get

$$y = 2(-4) - 3 = -11$$

Therefore, the solution to the linear system of equations or the point of intersection of the two lines is $(-4,-11)$.

Problem 4:

Solve the following system:

$$3y = -9$$
$$y = 2x$$

We will use the *Substitution Method* because one variable in the second equation has a coefficient of 1.

Furthermore, the second equation is already solved for y.

Back-substituting into the equation $3y = -9$, we get

$$3(2x) = -9$$

$$6x = -9$$

$$x = -\tfrac{9}{6} = -\tfrac{3}{2}$$

Solving for the y-coordinate using the equation $y = 2x$, we get

$$y = 2(-\tfrac{3}{2}) = -3$$

Therefore, the solution to the linear system of equations or the point of intersection of the two lines is $(-\tfrac{3}{2},-3)$.

Problem 5:

Solve the following system:

$$5x + 6y = 2$$
$$3x - 3y = 10$$

In this case, we will use the *Addition Method* because none of the variables have a coefficient of 1.

Let's eliminate x by multiplying the first equation by 3 and the second equation by -5. Then we'll add the two new equations to find x.

$$
\begin{array}{rr}
15x + 18y = & 6 \\
+ \quad -15x + 15y = & -50 \\
\hline
33y = & -44 \\
y = -\frac{44}{33} = -\frac{4}{3} &
\end{array}
$$

To find the y-coordinate of the point of intersection of the two lines, we back-substitute the value found for ***y*** into the equation $3x - 3y = 10$ to get

$$3x - 3(-\tfrac{4}{3}) = 10$$
$$3x + 4 = 10$$
$$x = 2$$

The solution to the system of linear equations or their point of intersection is $(2, -\frac{4}{3})$.

Problem 6:

Solve the following system.

$$8x = 2y + 8$$
$$3y = 12x - 12$$

First we must arrange the system so that the variable terms appear on one side of the equation and constants on the other side. Then we will use the *Addition Method* because none of the variables have a coefficient of ***1***. However, note that we could have divided the first equation by ***2*** and the second one by ***3*** to produce variables with coefficient ***1***.

$$
\begin{array}{rr}
8x - 2y = & 8 \\
-12x + 3y = & -12
\end{array}
$$

Let's eliminate ***y*** by multiplying the first equation by ***3*** and the second equation by ***2***. Then we'll add the two new equations to find ***x***.

$$
\begin{array}{rr}
24x - 6y = & 24 \\
+ \quad -24x + 6y = & -24 \\
\hline
0 = & 0
\end{array}
$$

Since we not only eliminated ***y*** but also ***x***, we have to conclude that this *System of Linear Equations* **has infinitely many solutions**. Graphically, you will find that both equations have the same graph. If you divide the first equation by ***2*** and the second equation by ***-3*** you can convince yourself of this!

Problem 7:

A grocer plans to mix candy that sells for $1.20 a pound with candy that sells for $2.40 a pound to get a mixture that he plans to sell for $1.65 a pound. How much of the $1.20 and $2.40 candy should he use if he wants 80 pounds of the mix?

Here we have enough information to make two equation in two variables. Let's call the candy that sells for $1.20 per pound x and the candy that sells for $2.40 per pound y.

The first equation is an income equation: $1.20x + 2.40y = 1.65(80)$

The second equation shows the total number of pounds in the mixture: $x + y = 80$

Therefore, we are solving the following system.

$$1.20x + 2.40y = 132$$

$$x + y = 80$$

We'll divide the first equation by -1.20 and then use the *Addition Method*.

$$-x - 2y = -110$$

$$\underline{x + y = 80}$$

$$-y = -30$$

$$y = 30$$

To find the value of x, we back-substitute the value found for y into the equation $x + y = 80$ to get

$$x + 30 = 80$$

$$x = 50$$

The grocer needs 50 lb of candy that sells for $1.20 and 30 lb of candy that sells for $2.40 to make an 80-lb mixture of candy that sells for $1.65.

Problem 8:

A charity has been receiving donations of dimes and quarters. They have 94 coins in all. If the total value is $19.30, how many dimes and how many quarters do they have?

Here we again have enough information to make two equation in two variables. Let's call the number of dimes x and the number of quarters y.

The first equation is an income equation: $0.10x + 0.25y = 19.30$

The second equation shows the total number of coins: $x + y = 94$

Therefore, we are solving the following system.

$$0.10x - 0.25y = 19.30$$

$$x + y = 94$$

We'll divide the first equation by ***-0.10*** and then use the *Addition Method*.

$$\begin{aligned} -x - 2.5y &= -193 \\ \underline{x + \quad y} &= \underline{\quad 94} \\ -1.5y &= -99 \\ y &= 66 \end{aligned}$$

To find the value of x, we back-substitute the value found for y into the equation $x + y = 94$ to get

$$x + 66 = 94$$

$$x = 28$$

The charity has 28 dimes and 66 quarters.

Problem 9:

An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for $360 and $450 per months respectively. When all units are rented, the total monthly rent is $4,950. What is the number of one- and two bedroom apartments?

Here we again have enough information to make two equation in two variables. Let's call the number of one-bedroom apartments x and the number of two-bedroom apartments y.

The first equation is an income equation: $360x + 450y = 4950$

The second equation shows the total number of apartments: $x + y = 12$

Therefore, we are solving the following system.

$$360x + 450y = 4950$$

$$x + y = 12$$

We'll multiply the second equation by ***-360*** and then use the *Addition Method*.

$$\begin{array}{rcr} 360x + 450y &=& 4950 \\ \underline{-360x - 360y} &\underline{=}& \underline{-4320} \\ 90y &=& 630 \\ y &=& 7 \end{array}$$

To find the value of ***x***, we back-substitute the value found for ***y*** into the equation ***x + y = 12*** to get

$$x + 7 = 12$$

$$x = 5$$

The apartment building has 5 one-bedroom apartments and 7 two-bedroom apartments.