DETAILED SOLUTIONS AND CONCEPTS - INEQUALITIES

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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Linear Inequalities

They differ from linear equalities in that instead of an equal sign they contain the following symbols:

> means greater than
$\geq$ means greater than or equal to
< means less than
$\leq$ means less than or equal to

The **properties of linear inequality** are almost exactly like the ones for linear equalities, except for one very important difference:

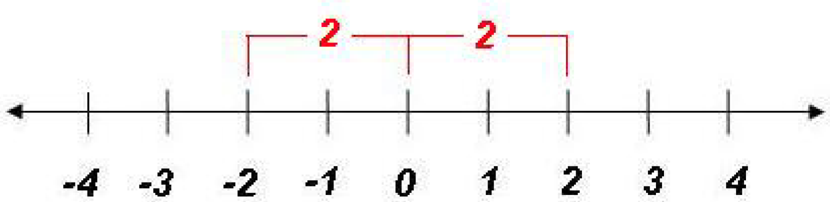
- When both sides of an inequality are **multiplied or divided by a negative numbe**r, the direction of the inequality symbol is reversed.
- Linear inequalities have infinitely many solutions. We call this a solution set.

Compound Linear Inequalities

They contain two inequality signs, for example, $-2 < 5x + 1 \leq 3$. When solving for the variable, we isolate ***x*** in the middle, applying the *Properties of Linear Inequality* to the right and left side of the inequality, as well as to the middle. The inequality signs can be any combination of **>**, **<**, $\leq$ or $\geq$.

Absolute Value

The *Absolute Value* expresses the distance between a number and ***0***. Let's look at the number line one more time.



We can see that the distance between ***0*** and ***2*** is ***2***. BUT, the distance between ***0*** and ***-2*** is also ***2***. We can express this as follows:

|2| = 2 means that the distance between ***0*** and ***2*** is ***2***. It is pronounced "the absolute value of two equals 2."

|-2| = 2 means that the distance between ***0*** and ***-2*** is also ***2***. It is pronounced "the absolute value of negative 2 equals 2."

Absolute Value Inequalities

They contain absolute value expression in addition to inequality signs, for example $|x+8|<5$. Absolute value inequalities have infinitely many solutions. We call this a solution set.

Solution Strategy:

- If necessary, isolate the absolute value expression on one side of the equal sign.
- Depending on the inequality sign, rewrite the inequality without absolute value bars as follows.

$|ax+b| \le c$ becomes $-c \le ax+b \le c$

$|ax+b| < c$ becomes $-c < ax+b < c$

$|ax+b| \ge c$ becomes $ax+b \ge c$ or $-(ax+b) \ge c$

$|ax+b| > c$ becomes $ax+b > c$ or $-(ax+b) > c$

The number ***c*** must be positive by definition of absolute value!!!

In the $\ge$ or > case, you have to solve both inequalities!!! The word "or" is a part of the formula and means one or the other or both.

Notation for a Continuous Solution Set

Parentheses ( ) : For example, in **Interval Notation**, a parenthesis next to a number indicates that the number is NOT included in the solution interval. Negative and positive infinity always start or end, respectively, with a parenthesis. See Table below.

Brackets [ ] : For example, in **Interval Notation**, a bracket next to a number indicates that the number is included in the solution interval. See Table below.

Braces { } : For example, in **Set-Builder Notation** we use braces. See Table below.

Solution	Interval Notation	Set-Builder Notation
$2 < x < 6$ The values of x are greater than 2 and smaller than 6.	$(2,6)$ an open interval	$\{ x \mid 2 < x < 6 \}$ Read the part in braces as "the set of all numbers x such that x is greater than 2 but less than 6."
$2 \leq x \leq 6$ The values of x are greater than or equal to 2 and smaller than or equal to 6.	$[2,6]$ a closed interval	$\{ x \mid 2 \leq x \leq 6 \}$
$2 \leq x < 6$ The values of x are greater than or equal to 2 and smaller than 6.	$[2,6)$ a half open interval	$\{ x \mid 2 \leq x < 6 \}$
$2 < x \leq 6$ The values of x are greater than 2 and smaller than or equal to 6.	$(2,6]$ a half open interval	$\{ x \mid 2 < x \leq 6 \}$
$x > 2$ The values of x are greater than 2 and there are infinitely many.	$(2,\infty)$ The positive infinity symbol ∞ ALWAYS has a parenthesis next to it!	$\{ x \mid x > 2 \}$
$x \geq 2$ The values of x are greater than or equal to 2 and there are infinitely many.	$[2,\infty)$	$\{ x \mid x \geq 2 \}$
$x < 2$ The values of x are smaller than 2 and there are infinitely many.	$(-\infty,2)$ The negative infinity symbol $-\infty$ ALWAYS has a parenthesis next to it!	$\{ x \mid x < 2 \}$
$x \leq 2$ The values of x are less than or equal to 2 and there are infinitely many.	$(-\infty,2]$	$\{ x \mid x \leq 2 \}$
All Real Numbers	$(-\infty,\infty)$	***{ x | x is a rational or an irrational number}***
All Real Numbers except -1 and 5	$(-\infty - 1) \cup (-1,5) \cup (5,\infty)$ Note: $\cup$ (Union) means one or the other or both solutions are possible.	$\{ x \mid x \neq -1 ,\ x \neq 5 \}$ Read the part in braces as "the set of all numbers x such that x is not equal to -1 and not equal to 5."
$x < 2$ but $x \neq 0$	$(-\infty,0) \cup (0,2)$	$\{ x \mid x < 2 ,\ x \neq 0 \}$

Problem 1:

Find the solution set for $3-5x \geq 13$ in *Interval Notation*. Then graph the solution set on the number line!

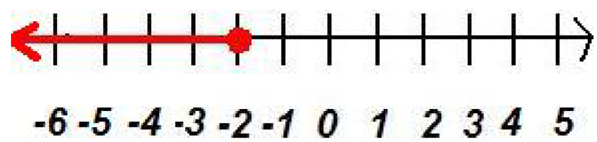
$-5x \geq 10$

Now we have to multiply both sides be the reciprocal of -5 or we can say that we must divide both sides by -5! **Note that the inequality sign changes direction when we divide or multiply by a negative number!**

$x \leq -2$

The solution set in *Interval Notation:* $(-\infty, -2]$

The graph of the solution set:



Please note that the graph contains a solid dot at -2 to indicate that -2 is included in the solution set! The arrow indicates that the solution set includes infinitely many numbers less than -2!

Problem 2:

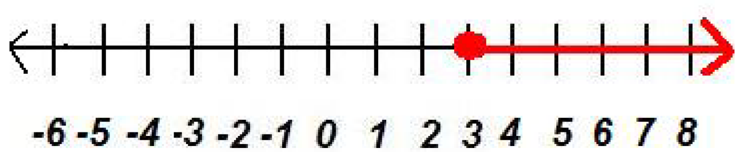
Find the solution set for $6x-15 \geq 3$ in *Interval Notation*. Then graph the solution set on the number line!

$6x \geq 18$

$x \geq 3$

The solution set in *Interval Notation*: $[3, \infty)$

The graph of the solution set:



Please note that 3 is included in the solution set. The arrow indicates that the solution set includes infinitely many numbers greater than 3!

Problem 3:

Find the solution set for $x - 9 < 5x + 7$ in *Interval Notation*. Then graph the solution set on the number line!

First, we will combine like terms as follows:

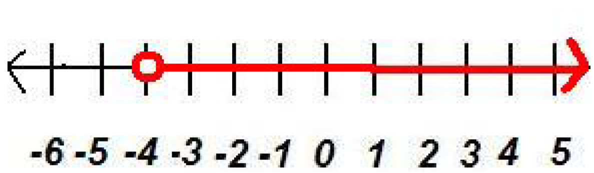
$-4x < 16$

Now we have to multiply both sides of the inequality by the **reciprocal of *-4***, which is the same as **dividing both sides by *-4***!

We get $x > -4$ **Note that the inequality sign changes direction when we divide or multiply by a negative number!**

The solution set in *Interval Notation*: $(-4, \infty)$.\

The graph of the solution set:



Please note that the graph contains a circle at -4 to indicate that -4 is **NOT** included in the solution set! The arrow indicates that the solution set includes infinitely many numbers greater than -4!

Problem 4:

Find the solution set for $-2 < 5x + 1 \leq 3$ in *Interval Notation*. Then graph the solution set on the number line!

Subtracting 1 from the left and right side of the inequality and from the middle, we get

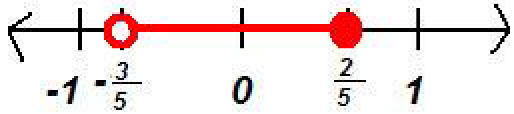
$-3 < 5x \leq 2$

Dividing 5 into the left and right side of the inequality, and into the middle, we get

$$-\frac{3}{5} < x \leq \frac{2}{5}$$

The solution set in *Interval Notation*: $(-\frac{3}{5}, \frac{2}{5}]$

The graph of the solution set:



Please note that the graph contains a circle at $-\frac{3}{5}$ to indicate that $-\frac{3}{5}$ is **NOT** included in the solution set and a solid dot at $\frac{2}{5}$ to indicated that $\frac{2}{5}$ is included in the solution set!

Problem 5:

Find the solution set for $\frac{3}{4} - x > \frac{7}{8}$ in *Interval Notation*. Then graph the solution set on the number line!

Again we want all constant terms on the right of the equal sign. Using the *Addition Axiom*, we get

$$-x > \frac{7}{8} - \frac{3}{4}$$

NOTE: Given fraction, it is a good idea to write the *Addition Axiom* out (at least on the right side) and not do mental calculations!

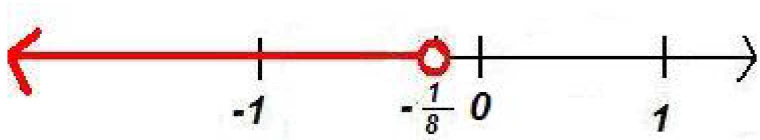
Now we have to combine like terms on the right using the ***LCD 8***. Then

$-x > \frac{7}{8} - \frac{3}{4}$ and $-x > \frac{1}{8}$

We actually now have to use the *Multiplication Axiom* to finish solving the equation. Since the coefficient of the variable must be ***+1*** for the equation to be solved, we must multiplied both sides of the equal sign by ***-1***, which is the reciprocal of ***-1***. Please not that the inequality sign will change direction!

and we find that $x < -\frac{1}{8}$.

The solution set in *Interval Notation*: $(-\infty, -\frac{1}{8})$.

The graph of the solution set: 

Please note that the graph contains a circle at $\frac{1}{8}$ to indicate that $\frac{1}{8}$ is **NOT** included in the solution set!

Problem 6:

Find the solution set for ***3x + 2(4 - 9x) -3(x - 3) + x < 0*** in *Interval Notation*.

First use the *Distributive Property of Multiplication* to eliminate parentheses.

This means that we can write the given equation in simplified form as follows:

3x + 8 - 18x - 3x + 9 + x < 0

Next, we'll combine like terms

-17x + 17 < 0

then we separate the variable and the constant

-17x < -17

and finally, we isolate the variable by multiplying both sides of the equality by the reciprocal of the coefficient of the variable to find that ***x > 1***.

This is ***(1, ∞)*** in *Interval Notation*. Please not that the inequality sign changed direction!

Problem 7:

Find the solution set for ***7 - (x - 8) ≤ 4x*** in *Interval Notation*.

First we us the *Distributive Property of Multiplication* to eliminate parentheses.

NOTE: Here you must know that we have a product ***-(x - 8)***, where one factor is the constant ***-1***. So we actually multiply each term in the second factor by ***-1*** as follows:

-(x - 8) = -1(x) -1(-8)

= -x + 8

This means that we can write our equation in simplified form as follows:

7 - x + 8 ≤ 4x

Next, we'll combine like terms on the left

$15 - x \leq 4x$

then we separate the variable and the constant by adding x to both sides to get

$15 \leq 5x$

and finally, we isolate the variable using the *Multiplication Axiom* to get $3 \leq x$.

It is customary in algebra to place the variable on the left side, therefore, we must write $x \geq 3$.

This is $[3, \infty)$ in *Interval Notation*.

Problem 8:

Find the solution set for $\frac{2}{3} \leq \frac{5-3x}{2} < \frac{3}{4}$ in *Interval Notation*.

With this type of inequality, it is best to clear fractions immediately. Simply multiply each part by the common denominator taking all given denominators into account.

The LCD is ***12*** !!!

$$12\left(\frac{2}{3}\right) \leq 12\left(\frac{5-3x}{2}\right) < 12\left(\frac{3}{4}\right)$$

Reducing fractions, we get

$$8 \leq 6(5-3x) < 9$$

and using the *Distributive Property*

$$8 \leq 30 - 18x < 9$$

$$-22 \leq -18x < -21$$

$$\frac{-22}{-18} \geq x > \frac{-21}{-18}$$

$$\frac{11}{9} \geq x > \frac{7}{6}$$

It is standard procedure in compound inequality to have all inequality signs point to the left, therefore, we rearrange our terms without changing the solution set

$$\frac{7}{6} < x \le \frac{11}{9}$$

The solution set in *Interval Notation* is $(\frac{7}{6}, \frac{11}{9}]$.

Problem 9:

Find the solution set for $|x-1| < 5$ in *Interval Notation*. Then graph the solution set on the number line!

By definition above, the "less than" case is rewritten as a compound inequality as follows

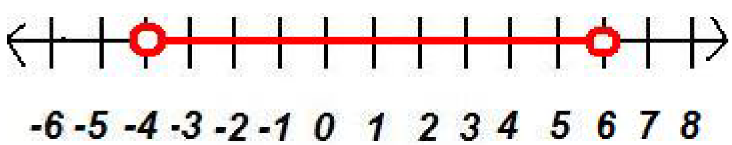
$$-5 < x - 1 < 5$$

Isolating the variable in the middle, we get

$$-4 < x < 6$$

The solution set in *Interval Notation*: $(-4,6)$

The graph of the solution set:



Please note that the graph contains a circle at -4 and 6 to indicate that -4 and 6 are **NOT** included in the solution set.

Problem 10:

Find the solution set for $3|4-2x| \le 6$ in *Interval Notation*. Then graph the solution set on the number line!

Before we apply the definition, we MUST first isolate the absolute value as follows

$$|4-2x| \le 2$$

Next, we will rewrite the "less than or equal to" case as a compound inequality according to the above definition

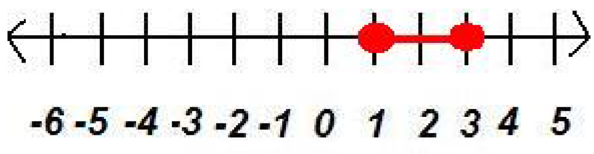
$$-2 \le 4 - 2x \le 2$$

$$-6 \le -2x \le -2$$

$3 \geq x \geq 1$ **Note that the inequality signs changed direction because we divided by a negative number!**

Changing the solution set to standard form, we get $1 \leq x \leq 3$.

The solution set in *Interval Notation*: $[1,3]$

The graph of the solution set: 

Please note that the graph contains a solid dot at 1 and 3 to indicate that 1 and 3 are included in the solution set.

Problem 11:

Find the solution set for $|5x+4|>1$ in *Interval Notation*. Then graph the solution set on the number line!

By definition above, the "greater than" case is rewritten as two inequalities as follows:

$$5x+4>1$$

$$5x>-3$$

$$x>-\frac{3}{5}$$

or

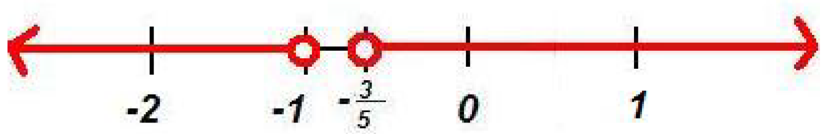
$$-(5x+4)>1$$

$$-5x-4>1$$

$$-5x>5$$

$$x<-1$$

The solution set in *Interval Notation:* $(-\infty,-1)\cup(-\frac{3}{5},\infty)$

The graph of the solution set: 

Please note that the graph contains a circle at $-\frac{3}{5}$ and -1 to indicate that $-\frac{3}{5}$ and -1 are **NOT** included in the solution set. The arrows indicate that the solution set includes infinitely many numbers less than -1 and greater than $-\frac{3}{5}$!

Problem 12:

Find the solution set for $|2x-1|\geq 3$ in *Interval Notation*. Then graph the solution set on the number line!

By definition above, the "greater than or equal" case is rewritten as two inequalities as follows:

$$2x-1\geq 3$$

$$2x\geq 4$$

and $x\geq 2$

or

$$-(2x-1)\geq 3$$

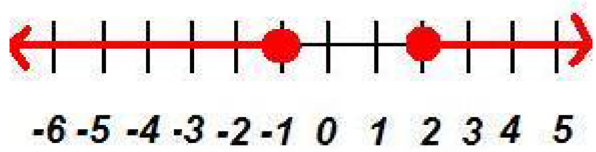
$$-2x+1\geq 3$$

$$-2x\geq 2$$

and $x\leq -1$ **Note that the inequality sign changes direction when we divide or multiply by a negative number!**

The solution set in *Interval Notation*: $(-\infty,-1] \cup [2,\infty)$

The graph of the solution set:



Please note that the graph contains a solid dot at -1 and 2 to indicate that -1 and 2 are included in the solution set. The arrows indicate that the solution set includes infinitely many numbers less than -1 and greater than 2!