



## DETAILED SOLUTIONS AND CONCEPTS - FORMULAS AND WORD PROBLEMS

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**PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!**

Formula rearrangement refers to isolating a letter term other than the one already isolated in the formula. Solving formulas in the manner often shortens our work when doing repeated formula evaluations. After we solve the formula for the desired variable, we rewrite the formula with the variable on the left side for convenience and for use in electronic spreadsheets.

### Problem 1:

Solve the "mark up" formula  $M = S - C$  for  $S$ .

To solve the equation above, we will use the *Addition Axiom* to isolate the variable  $S$  by "moving" all terms associated with  $S$  by addition away from the variable.

The variable associated with  $S$  by addition is  $C$ . Therefore, we must add  $C$  to both sides of the equation.

$$M + C = S - C + C$$

and  $M + C = S$

Finally, we rewrite the formula with  $S$  on the left

$$S = M + C$$

### Problem 2:

Solve the interest formula  $I = PRT$  for  $R$ . It stands for *Interest = Principal  $\times$  Rate  $\times$  Time*.

To solve the equation above, we will use the *Multiplication Axiom* to isolate the variable  $R$  by "moving" all terms associated with  $R$  by addition away from the variable.

The variables associated by multiplication with  $R$  are  $P \times T$  or  $PT$ . Therefore, we will

multiply both sides of the equation by its **reciprocal** \*\*, which is  $\frac{1}{PT}$ .

\*\* Interchanging the numerator and denominator of a fraction results in a fraction that is called the **reciprocal** of the original fraction. When a number is multiplied by its reciprocal, the product equals **1**.

$$\left(\frac{1}{PT}\right)(I) = \left(\frac{1}{PT}\right)(PRT)$$

Carrying out the multiplications, we get

$$\frac{I}{PT} = R \quad \text{which should be presented as} \quad R = \frac{I}{PT}.$$

### Problem 3:

Solve the formula  $P = 2(b + s)$  for  $b$ .

First, we use the *Distributive Property of Multiplication* to "open up" the parentheses.

$$P = 2b + 2s$$

To solve the equation above, we will use the *Addition Axiom* to isolate the variable  $b$  by "moving" all terms associated with  $b$  by addition away from the variable.

The variable associated with  $b$  by addition is  $2s$ . Therefore, we must subtract  $2s$  from both sides of the equation.

$$P - 2s = 2b + 2s - 2s$$

and

$$P - 2s = 2b$$

To solve the equation above, we will use the *Multiplication Axiom* to further isolate the variable  $b$ . The coefficient associated by multiplication with  $b$  is  $2$ . Therefore, we will multiply both sides of the equation by its reciprocal, which is  $\frac{1}{2}$ .

$$\left(\frac{1}{2}\right)(P - 2s) = \left(\frac{1}{2}\right)(2b)$$

Carrying out the multiplications, we get

$$\frac{P - 2s}{2} = b \quad \text{which should be presented as} \quad b = \frac{P - 2s}{2}.$$

#### Problem 4:

Solve the formula  $y = mx + b$  for  $m$ .

To solve the equation above, we will use the *Addition Axiom* to isolate the variable  $m$  by "moving" all terms associated with  $m$  by addition away from the variable.

The variable associated with  $m$  by addition is  $b$ . Therefore, we must subtract  $b$  from both sides of the equation.

$$y - b = mx + b - b$$

and

$$y - b = mx$$

To solve the equation above, we will use the *Multiplication Axiom* to further isolate the variable  $m$ . The variable associated by multiplication with  $m$  is  $x$ . Therefore, we will multiply both sides of the equation by its reciprocal, which is  $\frac{1}{x}$ .

$$\left(\frac{1}{x}\right)(y - b) = \left(\frac{1}{x}\right)mx$$

Carrying out the multiplications, we get

$$\frac{y - b}{x} = m \quad \text{which should be presented as} \quad m = \frac{y - b}{x} .$$

#### Problem 5:

Solve the Fahrenheit-Celsius conversion formula  $F = \frac{9}{5}C + 32$  for  $C$ .

To solve the equation above, we will use the *Addition Axiom* to isolate the variable  $C$  by "moving" all terms associated with  $C$  by addition away from the variable.

The constant associated with  $C$  by addition is  $32$ . Therefore, we must subtract  $32$  from both sides of the equation.

$$F - 32 = \frac{9}{5}C$$

To solve the equation above, we will use the *Multiplication Axiom* to further isolate the variable  $C$ . The coefficient associated by multiplication with  $C$  is  $\frac{9}{5}$ . Therefore, we will multiply both sides of the equation by its reciprocal, which is  $\frac{5}{9}$ .

$$\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\left(\frac{9}{5}C\right)$$

Carrying out the multiplications, we get the formula for converting from a Fahrenheit temperature  $F$  to a Celsius temperature  $C$ :

$$C = \frac{5}{9}(F - 32)$$

### Problem 6:

Solve the formula for calculating the circumference of a circle  $C = 2\pi r$  for  $r$ .

To solve the equation above, we will use the *Multiplication Axiom* to isolate the variable  $r$ . The coefficient associated by multiplication with  $r$  is  $2\pi$ . Therefore, we will multiply both

sides of the equation by its reciprocal, which is  $\frac{1}{2\pi}$ .

$$\left(\frac{1}{2\pi}\right)(C) = \left(\frac{1}{2\pi}\right)(2\pi r)$$

Carrying out the multiplications, we get  $r = \frac{C}{2\pi}$ .

## Word Problems - Translating Verbal Statements into Mathematical Statements

- Assign a letter to represent the missing number. In mathematics, the letter  $x$  is most commonly used. However, you may also use a letter that is more meaningful to you. For example, you may want to use  $w$  if you are asked to find a width or  $h$  if you are asked to find a height, etc.
- Identify key words or phrases that imply or suggest a specific operation.
- Translate words into symbols.

### KEY WORDS:

**Addition:** the sum of, plus, increased by, more than, added to, exceeds, longer, total, heavier, older, wider, taller, gain, greater than, more, gain

**Subtraction:** less than, decreased by, subtracted from, the difference between, diminished by, take away, reduced by, less, minus, shrinks, younger, lower, shorter, narrower, slower, loss

**Multiplication:** times (two times, three times, etc), multiply, of, the product of, multiplied by, twice, double, triple

**Division:** divide, divided by, divided into, how big is each part, how many parts can be made from

### Problem 7:

If a number is divided by 3 and then 2 is added, the result is 10. What is the number?

First we have to translate this sentence into "matherese". We have to give the number we are looking for a name. It is customary to assign a letter of the alphabet to it and the letter **x** is used most commonly.

Therefore, the phrase "a number is divided by 3" is translated as  $\frac{x}{3}$

The phrase "a number is divided by 3 and then 2 is added" is translated as  $\frac{x}{3} + 2$

The phrase "the result is 10" is translated as  $= 10$

Finally, we can write

$$\frac{x}{3} + 2 = 10$$

Let's solve this equation for **x**.

In this case, the coefficient of **x** is somewhat "disguised." What we need to know is that we can write

$$\frac{x}{3} \text{ as } \frac{1}{3}x$$

Now we can see that the coefficient is  $\frac{1}{3}$  so that we can write

$$\frac{1}{3}x + 2 = 10$$

Next, we will subtract 2 from both sides to get

$$\frac{1}{3}x = 8$$

Then  $\left(\frac{3}{1}\right)\left(\frac{1}{3}x\right) = \left(\frac{3}{1}\right)(8)$  and carrying out the multiplications, we get

$$x = 24$$

Hence, we found that the number we are looking for equals **24**.

### Problem 8:

Three times a number is 6 more than 30. What is the number?

Again, we have to translate this sentence into "matheresese". If we assign the letter  $x$  to the unknown number, the phrase "three times a number" is translated as  $3x$

The phrase "is 6 more than 30" is translated as  $= 6 + 30$

Finally, we can write

$$3x = 6 + 30$$

and  $3x = 36$ .

Let's solve this equation for  $x$ .

The coefficient associated by multiplication with  $x$  is  $3$ . Therefore, we either divide both sides by  $3$  or we will multiply both sides of the equation by its reciprocal, which is  $\frac{1}{3}$ .

$$\left(\frac{1}{3}\right)(3x) = \left(\frac{1}{3}\right)(36)$$

and  $x = 12$ .

Hence, we found that the number we are looking for equals  $12$ .

### Problem 9:

One Student 1 completed two more than 5 times as many homework assignments as Student 2. Together they completed 50 homework assignments. How many homework assignments did each student complete?

In this case, we'll let the number of homework assignments that Student 2 completed be equal to  $x$ .

Then  $5x + 2$  must be the number of homework assignments that Student 1 completed.

Together they completed  $(5x + 2) + x$  homework assignments. Since we know that the total number is  $50$ , we can write the following equation:

$$5x + 2 + x = 50$$

and combining like terms we get

$$6x + 2 = 50$$

Next, we will subtract 2 from both sides to get

$$6x = 48$$

The coefficient associated by multiplication with  $x$  is **6**. Therefore, we either divide both sides by **6** or we will multiply both sides of the equation by its reciprocal, which is  $\frac{1}{6}$ .

$$\left(\frac{1}{6}\right)(6x) = \left(\frac{1}{6}\right)(48)$$

and  $x = 8$ .

We found that Student 2 completed 8 homework assignments and Student 1 completed 5 (8) + 2 or 42 homework assignments.

### Problem 10:

Three parts totaling 27 lb are packaged for shipping. Two parts weigh the same. The third part weighs 3 lb less than each of the two equal parts. Find the weight of each part.

In this case, we'll let the weight of the two parts that weigh the same be equal to  $x$ .

Then  $x - 3$  must be the weight of the third part.

Together they weigh  $x + x + (x - 3)$ . Since we know that the total weight is **27**, we can write the following equation:

$$x + x + x - 3 = 27$$

and combining like terms we get

$$3x - 3 = 27$$

Next, we will add 3 to both sides to get

$$3x = 30$$

The coefficient associated by multiplication with  $x$  is **3**. Therefore, we either divide both sides by **3** or we will multiply both sides of the equation by its reciprocal, which is  $\frac{1}{3}$ .

$$\left(\frac{1}{3}\right)(3x) = \left(\frac{1}{3}\right)(30)$$

and  $x = 10$ .

We found that two parts weigh 10 lb each and the third part weighs 10 - 3 or 7 lb.

### Problem 11:

If 5 mL of water are added to a medicine, there are 45 mL in all. How many mL of medicine were used?

In this case, we'll let the volume of the medicine be equal to  $x$ .

$$\text{Then } x + 5 = 45$$

$$\text{and } x = 40$$

We found that 40 mL of medicine were used.

### Problem 12:

A student purchased a used physics book, a used Spanish book and a graphing calculator. If the calculator cost twice as much as the Spanish book and the physics book cost \$70, what was the cost of the calculator and the Spanish book? The total before tax was \$235.

In this case, we'll let the price of the Spanish book be equal to  $x$ . Then  $2x$  must be the cost of the calculator.

Together they cost  $2x + x$ . Since we know that the Physics book cost  $\$70$  and that the total cost was  $\$235$ . Therefore, we can write the following equation:

$$2x + x + 70 = 235$$

and combining like terms we get

$$3x + 70 = 235$$

Next, we will subtract 70 from both sides to get

$$3x = 165$$

The coefficient associated by multiplication with  $x$  is  $3$ . Therefore, we either divide both sides by  $3$  or we will multiply both sides of the equation by its reciprocal, which is  $\frac{1}{3}$ .

$$\left(\frac{1}{3}\right)(3x) = \left(\frac{1}{3}\right)(165)$$

and we get  $x = 55$ .

We found that the Spanish book cost \$55 and the calculator cost \$110.