



DETAILED SOLUTIONS AND CONCEPTS - INTRODUCTION TO ALGEBRA
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PLEASE NOTE THAT YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Vocabulary

Equation - In mathematics we use the symbol "=" to show that quantities are equal to each other.

For example, we write the statement "8 minus 2 equals 6" as $8 - 2 = 6$. This statement is called an *equation*.

Variables - Some equations contain unknown numbers, which are usually represented by letters of the alphabet. In mathematics we most often use the letters **x** and **y**. These letters are called *variables* or *unknowns*.

Algebraic Expression - A mathematical statement containing numbers and variables is called an *algebraic expression*. There is no equal sign!

For example, $2x + x + 5$.

Term - Mathematical expressions that are single variables, constants, products, or quotients. They are separated from each other by plus or minus signs.

For example,

in $\frac{6}{x + y} + x - 4$, the terms are $\frac{6}{x + y}$, x , and -4
in $2x + x + 5$, the terms are $2x$ and x and 5
in $3(a - b) + c$, the terms are $3(a - b)$ and c

Problem 1:

Identify the number of terms in each expression:

a. $x + 7$

There are two terms, x and 7 .

b. $3x - 2(x + 3)$

There are two terms, $3x$ and $-2(x + 3)$.

c. $\frac{7}{a + b} + \frac{1}{a - b}$

There are two terms, $\frac{7}{a + b}$ and $\frac{1}{a - b}$.

d. $\frac{6}{x + y}$

There is one term.

Multiplication Notation Conventions - When a term is the product of letters alone (such as xy or abc) or the product of a number and one or more letters (such as $2x$ or $-5ab$), parentheses or other symbols of multiplication are usually omitted.

For example, xy means x times y and $2x$ means 2 times x .

Constant - A term that contains only numbers.

For example, in $2x + x - 5$, the term 5 is called a constant.

Coefficient - If a product contains numbers and variables, the number factor is called *coefficient* or *numeric coefficient*.

For example, in $2x - 3x$, the coefficients are 2 and -3 .

Coefficient of 1 or -1 - When a variable term has no written numerical coefficient, the coefficient is understood to be 1 or -1 . It is customary not to write the coefficient of 1 and -1 .

For example, x is understood to mean $1x$ and $-x$ is understood to mean $-1x$.

Problem 2:

Identify the numerical coefficient of each term:

a. $3x$

The coefficient is 3 .

b. $-5xy$

The coefficient is -5 .

c. $\frac{x}{7}$

The coefficient is $\frac{1}{7}$.

d. $\frac{3x}{4}$

The coefficient is $\frac{3}{4}$.

e. abc

The coefficient is 1 .

f. $-xy$

The coefficient is -1 .

Solving the Equation - Finding the value of the variable(s).

Isolating the Variable - When we are asked to solve an equation, our goal will always be to "isolate" the variable on one side of the equal sign.

"Isolating" means that ultimately we want the variable to be by itself on one side of the equal sign and have a coefficient of positive 1 .

Distributive Property of Multiplication

It states that if you have a product of two factors, where one factor consists of an algebraic expression enclosed in parentheses, you can multiply each term of the algebraic expression with the other factor.

For example, given the product $2(4 - 9x)$, we have two factors. One is the constant 2 and the other one is the algebraic expression $4 - 9x$.

According to the *Distributive Property of Multiplication* we can do the following:

$$\begin{aligned}2(4 - 9x) &= 2(4) + 2(-9x) \\ &= 8 - 18x\end{aligned}$$

Verifying that an Equation is True

- Find the value of the expression on each side of the equal sign.
- Compare the values for each side.

If the values for each side are equal, the equation is true.

If the values for each side are not equal, the equation is not true.

Problem 3:

Given $y + 5 = 11$, does $y = 6$ make the equation true? Answer Yes or No

Yes, since $6 + 5$ equals 11 .

Problem 4:

Given $-9 + R = 3$, does $R = 10$ make the equation true? Answer Yes or No.

No, since $-9 + 10$ equals 1 and not 3 .

In this case the constant associated with the variable by addition is a **negative 9**.

Therefore, we must add a **positive 9** to both sides of the equation.

Problem 5:

Given $3 = \frac{3}{2}x$, does $x = 2$ make the equation true? Answer Yes or No.

Yes, since $\frac{3}{2}(2)$ equals 3 .

Problem 6:

What is the value of the expression $5x^2 + 7xy - y^2$, when $x = 2$ and $y = -4$?

$$5(2)^2 + 7(2)(-4) - (-4)^2$$

Using the Order of Operation, we are going to evaluate the exponential expressions first.

$$\begin{aligned} 5(4) + 7(2)(-4) - (16) &= 20 - 56 - 16 \\ &= -52 \end{aligned}$$

Adding and Subtracting of Terms

Terms that have the *same variable raised to the same power* are called **like terms**.

For example, in $2x + x - 5$, the like terms are $2x$ and x .

- We can only add and subtract **like terms**. Specifically, we add and subtract their *coefficients* only.

For example, $2x + x - 5$ can be simplified to $(2 + 1)x - 5 = 3x - 5$.

Remember when you don't see a coefficient it is assumed to be 1.

Please note, that $3x - 5$ cannot be further simplified because one term contains a variable and the other term is a constant.

- In algebraic expressions containing more than one variable, the variable closest to the letter **A** in the alphabet is stated first and then the others in alphabetic order.

For example, it is customary to write $-3a + b$ instead of $b - 3a$. Although, both answers are correct.

- In products containing two variables, the variable closest to the letter **A** in the alphabet is stated first and then the others in alphabetic order.

For example, we customarily say xy and not yx .

Problem 7:

Simplify $3a - 7a + a$.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we are asked to "simplify" instead of to adding and subtracting like terms.

$$3a - 7a + a = (3 - 7 + 1)a$$

$$= -3a \quad \text{Remember when you don't see a coefficient it is assumed to be 1!}$$

Problem 8:

Simplify $-8x + y - 3y$.

Note that we can only add and subtract **like terms**. That is, terms that have the *same variable raised to the same power*.

$$\begin{aligned} -8x + y - 3y &= -8x + (1 - 3)y \\ &= -8x - 2y \end{aligned}$$

Problem 9:

Simplify $3x + 8 - x + 4 - x - 2$.

This time we are going to simplify the expression mentally. To avoid confusion it is a good idea to make a check mark above any term that has been added to the total.

$$\begin{array}{ccccccc} \checkmark & & \checkmark & & \checkmark & & \\ 3x + 8 - x + 4 - x - 2 & = & x + 10 \end{array}$$

Problem 10:

Simplify $2xy + 7yx$.

In this case, you have to know that xy and yx are the same product.

Therefore, the terms in our sum are like terms and can be added to get $9xy$.

Please note that usually the factor with a variable closest to the letter *A* in the alphabet is stated first and then the others in alphabetic order.

Problem 11:

Simplify $-3(2x + 5y - 6) - (5y - 2)$.

First, we are going to use the *Distributive Property of Multiplication* "to open up" the parentheses.

$$-3(2x + 5y - 6) - (5y - 2) = -6x - 15y + 18 - 5y + 2$$

Again we are going to simplify the resulting expression mentally.

$$\begin{array}{ccccccc} & \checkmark & & \checkmark & & & \\ -6x - 15y + 18 - 5y + 2 & = & -6x - 20y + 20 \end{array}$$

Problem 12:

Simplify $8(x + 2y) - 3(7x - 3y + 5)$.

First, we are going to use the *Distributive Property of Multiplication* "to open up" the parentheses.

$$8(x + 2y) - 3(7x - 3y + 5) = 8x + 16y - 21x + 9y - 15$$

Again we are going to simplify the resulting expression mentally.



$$8x + 16y - 21x + 9y - 15 = -13x + 25y - 15$$

Problem 13:

Simplify $\frac{5a}{6} - \frac{a}{4} + \frac{3a}{16}$.

Please note that this expression can also be written as follows: $\frac{5}{6}a - \frac{1}{4}a + \frac{3}{16}a$

All fractions have a different denominator. Before we can simplify, we will have to make the denominators the same. A little guess work will tell us that the Least Common Denominator (LCD) must be 48. However, if you want, you can be more "scientific"!!! See below.



Finding the Least Common Denominator:

Since **6**, **4**, and **16** are NOT prime numbers, the LCD should not be found by calculating their product.

NOTE: Certainly, **6(4)(16) = 384** is a common denominator, but not necessarily the least common denominator. When adding and subtracting fractions we always try to find the least common denominator mainly to keep the calculations manageable. This is especially important when adding/subtracting fractions by hand.

To find the least common denominator (LCD) we will first have to use the concept of **Prime Factorization**. That is, we'll factor each denominator into its prime factors. In our case, we have

$$6 = 2 \cdot 3$$

$$4 = 2 \cdot 2$$

$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

Observe that the factor **3** is listed once. However, the factor **2** is listed once, twice, and four times.

For the LCD, we must always create a product using the largest listing of of each prime number taking into account ALL denominators.

Therefore, we must form a product containing the largest listing of 2 and 3.
That is,

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$$

Now we have to change each fraction to an equivalent fraction using the least common denominator (LCD).

$$\frac{5}{6} = \frac{5}{6} \cdot \frac{8}{8} = \frac{40}{48} \quad \text{and} \quad \frac{1}{4} = \frac{1}{4} \cdot \frac{12}{12} = \frac{12}{48} \quad \text{and} \quad \frac{3}{16} = \frac{3}{16} \cdot \frac{3}{3} = \frac{9}{48}$$

Finally, we can write the following:

$$\begin{aligned} \frac{40a}{48} - \frac{12a}{48} + \frac{9a}{48} &= \frac{(40 - 12 + 9)a}{48} \\ &= \frac{37a}{48} \text{ or } \frac{37}{48}a \end{aligned}$$

This is a proper fraction reduced to lowest terms!

Translating Verbal Statements into Mathematical Statements

- Assign a letter to represent the missing number. In mathematics, the letter **x** is most commonly used. However, you may also use a letter that is more meaningful to you. For example, you may want to use **w** if you are asked to find a width or **h** if you are asked to find a height, etc.
- Identify key words or phrases that imply or suggest a specific operation.
- Translate words into symbols.

KEY WORDS:

Addition: the sum of, plus, increased by, more than, added to, exceeds, longer, total, heavier, older, wider, taller, gain, greater than, more, gain

Subtraction: less than, decreased by, subtracted from, the difference between, diminished by, take away, reduced by, less, minus, shrinks, younger, lower, shorter, narrower, slower, loss

Multiplication: times (two times, three times, etc), multiply, of, the product of, multiplied by, twice, double, triple

Division: divide, divided by, divided into, how big is each part, how many parts can be made from

Problem 14:

Write the following as a mathematical statement: **5 more than a number is 12**

"more than" means addition

"a number" is what we are finding and we will call it **x**

"is" means equals

$$5 + x = 12$$

Problem 15:

Write the following as a mathematical statement: **4 times the difference of a number and 3 is 15**

"4 times" means multiplication

"difference" means subtraction

"a number" is what we are finding and we will call it **x**

"is" means equals

$$4(x - 3) = 15$$

We had to use parentheses because we were asked to find "4 times the difference".

Problem 16:

Write the following as a mathematical statement: **the sum of 12, 7, and a third number is 17**

"sum" means addition

"a number" is what we are finding and we will call it **x**

"is" means equals

$$12 + 7 + x = 17$$

Problem 17:

If the sum of three numbers is **60** and one of the numbers is **x** , what is the sum of the other two?

The sum of the other two numbers must be **$60 - x$**

Problem 18:

If C represents the number of cucumbers purchased at 20 cents each and T represents the number of tomatoes purchased at 5 cents each, write an expression for the total purchase amount in cents.

We multiply the number of cucumbers by the price per cucumber in cents: $20C$

We multiply the number of tomatoes by the price per tomato in cents: $5T$

Therefore, the total purchase price in cents must be $20C + 5T$